Package 'skellam'

April 7, 2025

Version 0.2.4

Date 2025-04-05

Title Densities and Sampling for the Skellam Distribution

Description Functions for the Skellam distribution, including: density (pmf), cdf, quantiles and regression.

URL https://github.com/monty-se/skellam

License GPL (>= 2)

Imports stats

Suggests knitr, rmarkdown

VignetteBuilder knitr

RoxygenNote 7.3.2

Encoding UTF-8

Enhances VGAM

BuildVignettes true

Repository CRAN

NeedsCompilation no

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Date/Publication 2025-04-07 06:40:02 UTC

Contents

skellam *The Skellam Distribution*

Description

Density, distribution function, quantile function, and random generation for the Skellam distribution.

Usage

```
dskellam(x, lambda1, lambda2 = lambda1, log = FALSE)

dskellam.sp(x, lambda1, lambda2 = lambda1, log = FALSE)

pskellam(q, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)

pskellam.sp(q, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)

qskellam(p, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)

rskellam(n, lambda1, lambda2 = lambda1)
```

Arguments

x, q	For functions dskellam, dskellam.sp, and pskellam.sp: a numeric vector of quantiles.
lambda1, lambda2	Numeric vectors of (non-negative) means; lambda2 defaults to lambda1 if not provided.
log, log.p	Logical; if TRUE, returns the logarithm of the computed value.
lower.tail	Logical; if TRUE (default), returns $P(X \leq x)$; otherwise, returns $P(X > x)$.
p	For qskellam: a numeric vector of probabilities.
n	For rskellam: a non-negative integer specifying the number of observations.

Details

The Skellam distribution describes the difference between two independent Poisson random variables. This documentation covers:

Density:

```
dskellam(x, lambda1, lambda2 = lambda1, log = FALSE)
```

Distribution Function:

```
pskellam(q, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

Quantile Function:

```
qskellam(p, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

Random Generation:

```
rskellam(n, lambda1, lambda2 = lambda1)
```

Saddlepoint Approximations:

```
dskellam.sp(x, lambda1, lambda2 = lambda1, log = FALSE)
pskellam.sp(q, lambda1, lambda2 = lambda1, lower.tail = TRUE, log.p = FALSE)
```

If Y_1 and Y_2 are Poisson variables with means μ_1 and μ_2 and correlation ρ, then $X = Y_1 - Y_2$ is Skellam with parameters:

$$\lambda_1 = \mu_1 - \rho\sqrt{\mu_1\mu_2}$$

$$\lambda_2 = \mu_2 - \rho\sqrt{\mu_1\mu_2}$$

The density is given by:

$$I(2\sqrt{\lambda_1\lambda_2}, |x|)(\lambda_1/\lambda_2)^{x/2}\exp(-\lambda_1 - \lambda_2)$$

where $I(y, \nu)$ is the modified Bessel function of the first kind.

Value

- `dskellam` returns the (log) density.
- `pskellam` returns the (log) cumulative distribution function.
- `qskellam` returns the quantile function.
- `rskellam` generates random deviates.

Invalid `lambda` values will return `NaN` with a warning.

Note

The **VGAM** package also provides Skellam functions. This implementation offers a broader working range, correct handling when one rate parameter is zero, enhanced argument checking, and improved accuracy for $x < 0$ (in R versions prior to 2.9). Use `skellam::dskellam` or `VGAM::dskellam` to specify which implementation to use.

References

- Butler, R. (2007) *Saddlepoint Approximations with Applications*, Cambridge University Press.
- Johnson, N. L. (1959) On an extension of the connection between Poisson and χ^2 distributions. *Biometrika* 46, 352-362.
- Johnson, N. L., Kotz, S., & Kemp, A. W. (1993) *Univariate Discrete Distributions*, 2nd ed., John Wiley and Sons.
- Skellam, J. G. (1946) The frequency distribution of the difference between two Poisson variates. *Journal of the Royal Statistical Society, Series A* 109(3), 296.
- Strackee, J. & van der Gon, J. J. D. (1962) The frequency distribution of the difference between two Poisson variates. *Statistica Neerlandica* 16(1), 17-23.
- Wikipedia: https://en.wikipedia.org/wiki/Skellam_distribution

Examples

```
# Compare with Poisson when one lambda = 0
dskellam(0:10, 5, 0)
dpois(0:10, 5)

# Both lambdas non-zero
dskellam(c(-1,1), c(12,10), c(10,12))
pskellam(c(-1,0), c(12,10), c(10,12))

# Quantile function
qskellam(c(0.05, 0.95), 3, 4)

# Random generation
rskellam(10, 8.5, 10.25)
```

---

skellam.mle *Maximum Likelihood Estimation for the Skellam Distribution*

Description

Estimates the parameters of a Skellam distribution using maximum likelihood.

Usage

```
skellam.mle(x)
```

Arguments

x	A vector of integers (positive or negative).

Details

Instead of having to maximize the log-likelihood with respect to both parameters (λ_1 and λ_2), the function maximizes with respect to λ_2 while setting $\lambda_1 = \lambda_2 + \bar{x}$. This approach improves computational efficiency. The optimization is performed using `nlm` as it proved faster than `optimise`.

Value

A list with components:

iters Number of iterations required by `nlm`.

loglik Maximized log-likelihood value.

param Estimated parameters ($\hat{\lambda}_1$, $\hat{\lambda}_2$).

Author(s)

Michail Tsagris

References

- Butler, R. (2007) *Saddlepoint Approximations with Applications*, Cambridge University Press.
- Johnson, N. L. (1959) On an extension of the connection between Poisson and χ^2 distributions. *Biometrika* **46**, 352-362.
- Johnson, N. L.; Kotz, S.; Kemp, A. W. (1993) *Univariate Discrete Distributions*, 2nd ed., John Wiley and Sons.
- Skellam, J. G. (1946) The frequency distribution of the difference between two Poisson variates belonging to different populations. *Journal of the Royal Statistical Society, Series A* **109**(3), 296.
- Strackee, J.; van der Gon, J. J. D. (1962) The frequency distribution of the difference between two Poisson variates. *Statistica Neerlandica* **16**(1), 17-23.
- Abdulhamid, A. A.; Maha, A. O. (2010) On The Poisson Difference Distribution Inference and Applications. *Bulletin of the Malaysian Mathematical Sciences Society* **33**(1), 17-45.
- Wikipedia: Skellam distribution https://en.wikipedia.org/wiki/Skellam_distribution

Examples

```
# Basic example
x1 <- rpois(1000, 10)
x2 <- rpois(1000, 6)
x <- x1 - x2
skellam.mle(x)

# Larger sample size
x1 <- rpois(10000, 10)
x2 <- rpois(10000, 6)
x <- x1 - x2
skellam.mle(x)
```

`skellam.reg` *Skellam Regression*

Description

Fits a regression model assuming a Skellam distribution for the response variable.

Usage

```
skellam.reg(y, x)
```

Arguments

`y`	A vector of integers (positive or negative)
`x`	A matrix, vector or data.frame of covariates

Details

The function uses an exponential link function to ensure positive values for both rate parameters (λ_1 and λ_2). Optimization is performed using `nlm`.

Value

A list with components:

loglik Maximized log-likelihood value

param1 Matrix for λ_1 parameters:

- Column 1: Estimated coefficients
- Column 2: Standard errors
- Column 3: t-values (coef/se)
- Column 4: p-values (Wald test)

param2 Matrix for λ_2 parameters (same structure as param1)

Author(s)

Michail Tsagris

References

- Skellam, J. G. (1946) The frequency distribution of the difference between two Poisson variates belonging to different populations. *Journal of the Royal Statistical Society, Series A* **109**(3), 296.
- Strackee, J.; van der Gon, J. J. D. (1962) The frequency distribution of the difference between two Poisson variates. *Statistica Neerlandica* **16**(1), 17-23.
- Karlis D. and Ntzoufras I. (2009) *Analysis of sports data using bivariate Poisson models*. IMA Conference Presentation. http://www2.stat-athens.aueb.gr/~jbn/papers/files/20_Karlis_Ntzoufras_2009_IMA_presentation_handouts_v01.pdf

Examples

```
set.seed(0)
x <- rnorm(100)
y1 <- rpois(100, exp(1 + 1 * x))
y2 <- rpois(100, exp(-1 + 1 * x))
y <- y2 - y1
skellam.reg(y, x)
```

Index