Package 'pbkrtest'

April 28, 2025

Version 0.5.4

Title Parametric Bootstrap, Kenward-Roger and Satterthwaite Based
Methods for Test in Mixed Models

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Description Computes p-values based on (a) Satterthwaite or
Kenward-Rogers degree of freedom methods and (b) parametric bootstrap
for mixed effects models as implemented in the 'lme4'
package. Implements parametric bootstrap test for generalized linear
mixed models as implemented in 'lme4' and generalized linear
models. The package is documented in the paper by Halekoh and
Højsgaard, (2012, <doi:10.18637/jss.v059.i09>). Please see
'citation(``pbkrtest'')' for citation details.

URL https://people.math.aau.dk/~sorenh/software/pbkrtest/

Depends R (>= 4.2.0), lme4 (>= 1.1.31)

Imports broom, dplyr, MASS, methods, numDeriv, Matrix (>= 1.2.3), doBy
(>= 4.6.22)

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NeedsCompilation no

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Repository CRAN

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Contents

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anovax	*anova like function*

Description

anova like function

print anovax object

Usage

```
anovax(object, ..., test = "x2", control = list(nsim = 1000, cl = NULL))

## S3 method for class 'lmerMod'
anovax(object, ..., test = "x2", control = list(nsim = 1000, cl = NULL))

## Default S3 method:
anovax(object, ..., test = "x2", control = list(nsim = 1000, cl = NULL))

## S3 method for class 'anovax'
print(x, ...)
```

Arguments

`object`	A model object object
`...`	further arguments
`test`	A character string
`control`	A list controling simulations, only relevant for parametric bootstrapping.
`x`	anovax object

Author(s)

Søren Højsgaard

Examples

```
lmm1 <- lmer(sugpct ~ block + sow + harvest + (1|block:harvest), data=beets)
lmm0 <- update(lmm1, .~. - sow)
anovax(lmm1, .~. - harvest, test="KR")
anovax(lmm1, .~. - harvest, test="SAT")
## anovax(lmm1, .~. - harvest, test="PB", control=list(nsim=50, cl=1))

anovax(lmm1, test="KR")
anovax(lmm1, test="SAT")
anovax(lmm1, test="PB", control=list(nsim=50, cl=1))
```

---

anovax_list	*Various different tests for model comparison*

Description

Various different tests for model comparison

Usage

```
anovax_list(
  object,
  object2,
  test = c("x2", "kr", "sat", "pb"),
  control = list(nsim = 1000)
)
```

Arguments

`object`	Model object
`object2`	Model object or equivalent way of specifying a submodel of lmm1
`test`	A vector with the various test types.
`control`	A list controlling the model comparions.

Value

Dataframe with results of the various tests

Author(s)

Søren Højsgaard

---

comodex   *Model comparison*

Description

Wrapper for functions KRmodcomp, SATmodcomp, PBmodcomp, X2modcomp

Usage

```
comodex(
  largeModel,
  smallModel,
  test = "x2",
  control = list(),
  details = 0,
  ...
)

## S3 method for class 'lmerMod'
comodex(
  largeModel,
  smallModel,
  test = "x2",
  control = list(),
  details = 0,
  ...
)

## Default S3 method:
comodex(
  largeModel,
  smallModel,
  test = "x2",
  control = list(),
  details = 0,
  ...
)
```

Arguments

largeModel	A model object
smallModel	A model object, a formula or a restriction matrix
test	A character string
control	A list controlling the model comparions.
details	should details be printed
...	Additional arguments to be passed on to other methods

Author(s)

Søren Højsgaard

Examples

```
(lmm0 <- lmer(Reaction ~ (Days|Subject), sleepstudy))
(lmm1 <- lmer(Reaction ~ Days + (Days|Subject), sleepstudy))
(lmm2 <- lmer(Reaction ~ Days + I(Days^2) + (Days|Subject), sleepstudy))

lm1 <- lm(dist ~ speed + I(speed^2), data=cars)
lm0 <- lm(dist ~ speed, data=cars)

comodex(lmm2, lmm1, test="x2")
comodex(lmm2, lmm1, test="kr")
comodex(lmm2, lmm1, test="sat")
comodex(lmm2, lmm1, test="PB", control=list(nsim=50, cl=1))
comodex(lmm2, .~. - I(Days^2))

comodex(lm1, lm0)
comodex(lm1, lm0, test="pb", control=list(nsim=50, cl=1))
```

---

compare_column_space   *Compare column spaces*

Description

Compare column spaces of two matrices

Usage

```
compare_column_space(X1, X2)
```

Arguments

X1, X2	matrices with the same number of rows

Value

- -1 : Either C(X1)=C(X2), or the spaces are not nested.
- 0 : C(X1) is contained in C(X2)
- 1 : C(X2) is contained in C(X1)

Examples

```
A1 <- matrix(c(1,1,1,1,2,3), nrow=3)
A2 <- A1[, 1, drop=FALSE]

compare_column_space(A1, A2)
compare_column_space(A2, A1)
compare_column_space(A1, A1)
```

---

data-beets   *Sugar beets data*

Description

Yield and sugar percentage in sugar beets from a split plot experiment. The experimental layout was as follows: There were three blocks. In each block, the harvest time defines the "whole plot" and the sowing time defines the "split plot". Each plot was $25m^2$ and the yield is recorded in kg. See 'details' for the experimental layout. The data originates from a study carried out at The Danish Institute for Agricultural Sciences (the institute does not exist any longer; it became integrated in a Danish university).

Usage

```
beets
```

Format

A dataframe with 5 columns and 30 rows.

Details

```
Experimental plan
Sowing times          1        4. april
                      2       12. april
                      3       21. april
                      4       29. april
                      5       18. may
Harvest times         1        2. october
                      2       21. october
Plot allocation:
            Block 1     Block 2     Block 3
```

```
          +-----------|-----------|-----------+
  Plot    | 1 1 1 1 1 | 2 2 2 2 2 | 1 1 1 1 1 | Harvest time
  1-15    | 3 4 5 2 1 | 3 2 4 5 1 | 5 2 3 4 1 | Sowing time
          |-----------|-----------|-----------|
  Plot    | 2 2 2 2 2 | 1 1 1 1 1 | 2 2 2 2 2 | Harvest time
  16-30   | 2 1 5 4 3 | 4 1 3 2 5 | 1 4 3 2 5 | Sowing time
          +-----------|-----------|-----------+
```

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

Examples

```
data(beets)

beets$bh <- with(beets, interaction(block, harvest))
summary(aov(yield ~ block + sow + harvest + Error(bh), beets))
summary(aov(sugpct ~ block + sow + harvest + Error(bh), beets))
```

---

data-budworm   *Budworm data*

Description

Experiment on the toxicity to the tobacco budworm Heliothis virescens of doses of the pyrethroid trans-cypermethrin to which the moths were beginning to show resistance. Batches of 20 moths of each sex were exposed for three days to the pyrethroid and the number in each batch that were dead or knocked down was recorded. Data is reported in Collett (1991, p. 75).

Usage

```
budworm
```

Format

This data frame contains 12 rows and 4 columns:

sex: sex of the budworm.

dose: dose of the insecticide trans-cypermethrin (in micro grams).

ndead: budworms killed in a trial.

ntotal: total number of budworms exposed per trial.

Source

Collett, D. (1991) Modelling Binary Data, Chapman & Hall, London, Example 3.7

References

Venables, W.N; Ripley, B.D.(1999) Modern Applied Statistics with S-Plus, Heidelberg, Springer, 3rd edition, chapter 7.2

Examples

```
data(budworm)

## function to caclulate the empirical logits
empirical.logit<- function(nevent,ntotal) {
   y <- log((nevent + 0.5) / (ntotal - nevent + 0.5))
   y
}


# plot the empirical logits against log-dose

log.dose <- log(budworm$dose)
emp.logit <- empirical.logit(budworm$ndead, budworm$ntotal)
plot(log.dose, emp.logit, type='n', xlab='log-dose',ylab='emprirical logit')
title('budworm: emprirical logits of probability to die ')
male <- budworm$sex=='male'
female <- budworm$sex=='female'
lines(log.dose[male], emp.logit[male], type='b', lty=1, col=1)
lines(log.dose[female], emp.logit[female], type='b', lty=2, col=2)
legend(0.5, 2, legend=c('male', 'female'), lty=c(1,2), col=c(1,2))

## Not run:
* SAS example;
data budworm;
infile 'budworm.txt' firstobs=2;
input sex dose ndead ntotal;
run;

## End(Not run)
```

---

get_ddf_Lb   *Adjusted denominator degrees of freedom for linear estimate for linear mixed model.*

Description

Get adjusted denominator degrees freedom for testing Lb=0 in a linear mixed model where L is a restriction matrix.

Usage

```
get_Lb_ddf(object, L)

## S3 method for class 'lmerMod'
get_Lb_ddf(object, L)

get_ddf_Lb(object, Lcoef)

## S3 method for class 'lmerMod'
get_ddf_Lb(object, Lcoef)

Lb_ddf(L, V0, Vadj)

ddf_Lb(VVa, Lcoef, VV0 = VVa)
```

Arguments

`object`	A linear mixed model object.
`L`	A vector with the same length as `fixef(object)` or a matrix with the same number of columns as the length of `fixef(object)`
`Lcoef`	Linear contrast matrix
`V0, Vadj`	The unadjusted and the adjusted covariance matrices for the fixed effects parameters. The unadjusted covariance matrix is obtained with `vcov()` and adjusted with `vcovAdj()`.
`VVa`	Adjusted covariance matrix
`VV0`	Unadjusted covariance matrix

Value

Adjusted degrees of freedom (adjustment made by a Kenward-Roger approximation).

Author(s)

Søren Højsgaard, <sorenh@math.aau.dk>

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

See Also

KRmodcomp, vcovAdj, model2restriction_matrix, restriction_matrix2model

Examples

```
(fmLarge <- lmer(Reaction ~ Days + (Days|Subject), sleepstudy))
## removing Days
(fmSmall <- lmer(Reaction ~ 1 + (Days|Subject), sleepstudy))
anova(fmLarge, fmSmall)

KRmodcomp(fmLarge, fmSmall)  ## 17 denominator df's
get_Lb_ddf(fmLarge, c(0, 1)) ## 17 denominator df's

# Notice: The restriction matrix L corresponding to the test above
# can be found with
L <- model2restriction_matrix(fmLarge, fmSmall)
L
```

---

get_modcomp   *Extract (or "get") components from a* KRmodcomp *object.*

Description

Extract (or "get") components from a KRmodcomp object, which is the result of the KRmodcomp function.

Usage

```
getKR(
  object,
  name = c("ndf", "ddf", "Fstat", "p.value", "F.scaling", "FstatU", "p.valueU", "aux")
)

getSAT(object, name = c("ndf", "ddf", "Fstat", "p.value"))
```

Arguments

object	A KRmodcomp object, which is the result of the KRmodcomp function
name	The available slots. If name is missing or NULL then everything is returned.

Author(s)

Søren Højsgaard <sorenh@math.aau.dk>

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

See Also

KRmodcomp, PBmodcomp, vcovAdj

Examples

```
data(beets, package='pbkrtest')
lg <- lmer(sugpct ~ block + sow + harvest + (1|block:harvest),
             data=beets, REML=FALSE)
sm <- update(lg, .~. - harvest)
modcomp <- KRmodcomp(lg, sm)
getKR(modcomp, "ddf") # get denominator degrees of freedom.
```

---

internal   *Internal functions for the pbkrtest package*

Description

These functions are not intended to be called directly.

---

internal-pbkrtest   *pbkrtest internal*

Description

pbkrtest internal

---

kr-vcovAdj   *Adjusted covariance matrix for linear mixed models according to Kenward and Roger*

Description

Kenward and Roger (1997) describe an improved small sample approximation to the covariance matrix estimate of the fixed parameters in a linear mixed model.

Usage

```
vcovAdj(object, details = 0)

## S3 method for class 'lmerMod'
vcovAdj(object, details = 0)
```

Arguments

`object`	An `lmer` model
`details`	If larger than 0 some timing details are printed.

Value

`phiA`	the estimated covariance matrix, this has attributed P, a list of matrices used in `KR_adjust` and the estimated matrix W of the variances of the covariance parameters of the random effects
`SigmaG`	list: Sigma: the covariance matrix of Y; G: the G matrices that sum up to Sigma; `n.ggamma`: the number (called M in the article) of G matrices)

Note

If N is the number of observations, then the `vcovAdj()` function involves inversion of an $N \times N$ matrix, so the computations can be relatively slow.

Author(s)

Ulrich Halekoh <uhalekoh@health.sdu.dk>, Søren Højsgaard <sorenh@math.aau.dk>

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

Kenward, M. G. and Roger, J. H. (1997), *Small Sample Inference for Fixed Effects from Restricted Maximum Likelihood*, Biometrics 53: 983-997.

See Also

getKR, KRmodcomp, lmer, PBmodcomp, vcovAdj

Examples

```
fm1 <- lmer(Reaction ~ Days + (Days|Subject), sleepstudy)
class(fm1)

## Here the adjusted and unadjusted covariance matrices are identical,
## but that is not generally the case:

v1 <- vcov(fm1)
v2 <- vcovAdj(fm1, details=0)
v2 / v1

## For comparison, an alternative estimate of the variance-covariance
## matrix is based on parametric bootstrap (and this is easily
## parallelized):

## Not run:
```

```
nsim <- 100
sim <- simulate(fm.ml, nsim)
B <- lapply(sim, function(newy) try(fixef(refit(fm.ml, newresp=newy))))
B <- do.call(rbind, B)
v3 <- cov.wt(B)$cov
v2/v1
v3/v1

## End(Not run)
```

---

kr__modcomp	*F-test and degrees of freedom based on Kenward-Roger approximation*

Description

An approximate F-test based on the Kenward-Roger approach.

Usage

```
KRmodcomp(largeModel, smallModel, betaH = 0, details = 0)

## S3 method for class 'lmerMod'
KRmodcomp(largeModel, smallModel, betaH = 0, details = 0)
```

Arguments

largeModel	An lmer model
smallModel	An lmer model or a restriction matrix
betaH	A number or a vector of the beta of the hypothesis, e.g. L beta=L betaH. If smallModel is a model object then betaH=0.
details	If larger than 0 some timing details are printed.

Details

An F test is calculated according to the approach of Kenward and Roger (1997). The function works for linear mixed models fitted with the lmer() function of the lme4 package. Only models where the covariance structure is a linear combination (a weighted sum) of known matrices can be compared.

The smallModel is the model to be tested against the largeModel.

The largeModel is a model fitted with lmer(). A technical detail: The model must be fitted with REML=TRUE. If the model is fitted with REML=FALSE then the model is refitted with REML=TRUE before the p-values are calculated. Put differently, the user needs not worry about this issue.

The smallModel can be one of several things:

1. a model fitted with `lmer()`. It must have the same covariance structure as `largeModel`. Furthermore, its linear space of expectation must be a subspace of the space for `largeModel`.

2. a restriction matrix `L` specifying the hypothesis

$$L\beta = L\beta_H$$

   where `L` is a `k x p` matrix (there are k restrictions and p is the number of fixed effect parameters (the length of `fixef(largeModel)`) and `beta_H` is a p column vector.

3. A formula or a text string specifying what is to be removed from the larger model to form the smaller model.

Notice: if you want to test a hypothesis

$$L\beta = c$$

with a k vector c, a suitable β_H is obtained via $\beta_H = Lc$ where L_n is a g-inverse of L.

Notice: It cannot be guaranteed that the results agree with other implementations of the Kenward-Roger approach!

Author(s)

Ulrich Halekoh <uhalekoh@health.sdu.dk>, Søren Højsgaard <sorenh@math.aau.dk>

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

Kenward, M. G. and Roger, J. H. (1997), *Small Sample Inference for Fixed Effects from Restricted Maximum Likelihood*, Biometrics 53: 983-997.

See Also

getKR, lmer, vcovAdj, PBmodcomp, SATmodcomp

Examples

```
(fm0 <- lmer(Reaction ~ (Days|Subject), sleepstudy))
(fm1 <- lmer(Reaction ~ Days + (Days|Subject), sleepstudy))
(fm2 <- lmer(Reaction ~ Days + I(Days^2) + (Days|Subject), sleepstudy))

## Test for no effect of Days in fm1, i.e. test fm0 under fm1
KRmodcomp(fm1, "Days")
KRmodcomp(fm1, ~.-Days)
L1 <- cbind(0, 1)
KRmodcomp(fm1, L1)
KRmodcomp(fm1, fm0)
anova(fm1, fm0)

## Test for no effect of Days and Days-squared in fm2, i.e. test fm0 under fm2
```

```
KRmodcomp(fm2, "(Days+I(Days^2))")
KRmodcomp(fm2, ~. - Days - I(Days^2))
L2 <- rbind(c(0, 1, 0), c(0, 0, 1))
KRmodcomp(fm2, L2)
KRmodcomp(fm2, fm0)
anova(fm2, fm0)

## Test for no effect of Days-squared in fm2, i.e. test fm1 under fm2
KRmodcomp(fm2, "I(Days^2)")
KRmodcomp(fm2, ~. - I(Days^2))
L3 <- rbind(c(0, 0, 1))
KRmodcomp(fm2, L3)
KRmodcomp(fm2, fm1)
anova(fm2, fm1)
```

---

modcomp   *Compare two models*

Description

Compare two models

Usage

```
pb_modcomp(largeModel, smallModel, control = list())

kr_modcomp(largeModel, smallModel, control = list())

sat_modcomp(largeModel, smallModel, control = list())

x2_modcomp(largeModel, smallModel, control = list())
```

Arguments

largeModel, smallModel

                Two models

control   A list

---

model-coerce   *Conversion between a model object and a restriction matrix*

Description

Testing a small model under a large model corresponds imposing restrictions on the model matrix of the larger model and these restrictions come in the form of a restriction matrix. These functions converts a model to a restriction matrix and vice versa.

Usage

```
model2restriction_matrix(largeModel, smallModel, sparse = FALSE)

restriction_matrix2model(largeModel, L, REML = TRUE, ...)

make_model_matrix(X, L)

make_restriction_matrix(X, X2)
```

Arguments

largeModel, smallModel

              Model objects of the same "type". Possible types are linear mixed effects models and linear models (including generalized linear models)

sparse   Should the restriction matrix be sparse or dense?

L   A restriction matrix; a full rank matrix with as many columns as X has.

REML   Controls if new model object should be fitted with REML or ML.

...   Additional arguments; not used.

X, X2   Model matrices. Must have same number of rows.

Details

make_restriction_matrix Make a restriction matrix. If span(X2) is in span(X) then the corresponding restriction matrix L is returned.

Value

model2restriction_matrix: A restriction matrix. restriction_matrix2model: A model object.

Note

That these functions are visible is a recent addition; minor changes may occur.

Author(s)

Ulrich Halekoh <uhalekoh@health.sdu.dk>, Søren Højsgaard <sorenh@math.aau.dk>

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

See Also

PBmodcomp, PBrefdist, KRmodcomp

Examples

```
library(pbkrtest)
data("beets", package = "pbkrtest")
sug <- lm(sugpct ~ block + sow + harvest, data=beets)
sug.h <- update(sug, .~. - harvest)
sug.s <- update(sug, .~. - sow)

## Construct restriction matrices from models
L.h <- model2restriction_matrix(sug, sug.h); L.h
L.s <- model2restriction_matrix(sug, sug.s); L.s

## Construct submodels from restriction matrices
mod.h <- restriction_matrix2model(sug, L.h); mod.h
mod.s <- restriction_matrix2model(sug, L.s); mod.s

## Sanity check: The models have the same fitted values and log likelihood
plot(fitted(mod.h), fitted(sug.h))
plot(fitted(mod.s), fitted(sug.s))
logLik(mod.h)
logLik(sug.h)
logLik(mod.s)
logLik(sug.s)
```

---

pb-refdist   *Calculate reference distribution using parametric bootstrap*

Description

Calculate reference distribution of likelihood ratio statistic in mixed effects models using parametric bootstrap

Usage

```
PBrefdist(
  largeModel,
  smallModel,
  nsim = 1000,
  seed = NULL,
  cl = NULL,
  details = 0
)

## S3 method for class 'lm'
PBrefdist(
  largeModel,
  smallModel,
  nsim = 1000,
  seed = NULL,
```

```
  cl = NULL,
  details = 0
)

## S3 method for class 'merMod'
PBrefdist(
  largeModel,
  smallModel,
  nsim = 1000,
  seed = NULL,
  cl = NULL,
  details = 0
)

## S3 method for class 'gls'
PBrefdist(
  largeModel,
  smallModel,
  nsim = 1000,
  seed = NULL,
  cl = NULL,
  details = 0
)
```

Arguments

largeModel	A linear mixed effects model as fitted with the lmer() function in the **lme4** package. This model muse be larger than smallModel (see below).
smallModel	A linear mixed effects model as fitted with the lmer() function in the **lme4** package. This model muse be smaller than largeModel (see above).
nsim	The number of simulations to form the reference distribution.
seed	Seed for the random number generation.
cl	Used for controlling parallel computations. See sections 'details' and 'examples' below.
details	The amount of output produced. Mainly relevant for debugging purposes.

Details

The model object must be fitted with maximum likelihood (i.e. with REML=FALSE). If the object is fitted with restricted maximum likelihood (i.e. with REML=TRUE) then the model is refitted with REML=FALSE before the p-values are calculated. Put differently, the user needs not worry about this issue.

```
The argument 'cl' (originally short for 'cluster') is used for
controlling parallel computations. 'cl' can be NULL (default),
positive integer or a list of clusters.
```

Special care must be taken on Windows platforms (described below) but the general picture is this:

The recommended way of controlling cl is to specify the
component \code{pbcl} in options() with
e.g. \code{options("pbcl"=4)}.

If cl is NULL, the function will look at if the pbcl has been set
in the options list with \code{getOption("pbcl")}

If cl=N then N cores will be used in the computations. If cl is
NULL then the function will look for

Value

A numeric vector

Author(s)

Søren Højsgaard <sorenh@math.aau.dk>

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

See Also

PBmodcomp, KRmodcomp

Examples

```
data(beets)
head(beets)
beet0 <- lmer(sugpct ~ block + sow + harvest + (1|block:harvest), data=beets, REML=FALSE)
beet_no.harv <- update(beet0, . ~ . -harvest)
rd <- PBrefdist(beet0, beet_no.harv, nsim=20, cl=1)
rd
## Not run:
## Note: Many more simulations must be made in practice.

# Computations can be made in parallel using several processors:

# 1: On OSs that fork processes (that is, not on windows):
# -------------------------------------------------------

if (Sys.info()["sysname"] != "Windows"){
  N <- 2 ## Or N <- parallel::detectCores()

# N cores used in all calls to function in a session
  options("mc.cores"=N)
  rd <- PBrefdist(beet0, beet_no.harv, nsim=20)
```

```
# N cores used just in one specific call (when cl is set,
# options("mc.cores") is ignored):
  rd <- PBrefdist(beet0, beet_no.harv, nsim=20, cl=N)
}

# In fact, on Windows, the approach above also work but only when setting the
# number of cores to 1 (so there is to parallel computing)

# In all calls:
# options("mc.cores"=1)
# rd <- PBrefdist(beet0, beet_no.harv, nsim=20)
# Just once
# rd <- PBrefdist(beet0, beet_no.harv, nsim=20, cl=1)

# 2. On all platforms (also on Windows) one can do
# ---------------------------------------------
library(parallel)
N <- 2 ## Or N  <- detectCores()
clus <- makeCluster(rep("localhost", N))

# In all calls in a session
options("pb.cl"=clus)
rd <- PBrefdist(beet0, beet_no.harv, nsim=20)

# Just once:
rd <- PBrefdist(beet0, beet_no.harv, nsim=20, cl=clus)
stopCluster(clus)

## End(Not run)
```

pb__modcomp	*Model comparison using parametric bootstrap methods.*

Description

Model comparison of nested models using parametric bootstrap methods. Implemented for some commonly applied model types.

Usage

```
PBmodcomp(
  largeModel,
  smallModel,
  nsim = 1000,
  ref = NULL,
  seed = NULL,
  cl = NULL,
  details = 0
)
```

```
## S3 method for class 'merMod'
PBmodcomp(
  largeModel,
  smallModel,
  nsim = 1000,
  ref = NULL,
  seed = NULL,
  cl = NULL,
  details = 0
)

## S3 method for class 'lm'
PBmodcomp(
  largeModel,
  smallModel,
  nsim = 1000,
  ref = NULL,
  seed = NULL,
  cl = NULL,
  details = 0
)

seqPBmodcomp(largeModel, smallModel, h = 20, nsim = 1000, cl = 1)
```

Arguments

largeModel, smallModel

Two models

nsim	The number of simulations to form the reference distribution.
ref	Vector containing samples from the reference distribution. If NULL, this vector will be generated using PBrefdist().
seed	A seed that will be passed to the simulation of new datasets.
cl	A vector identifying a cluster; used for calculating the reference distribution using several cores. See examples below.
details	The amount of output produced. Mainly relevant for debugging purposes.
h	For sequential computing for bootstrap p-values: The number of extreme cases needed to generate before the sampling process stops.

Details

The model object must be fitted with maximum likelihood (i.e. with REML=FALSE). If the object is fitted with restricted maximum likelihood (i.e. with REML=TRUE) then the model is refitted with REML=FALSE before the p-values are calculated. Put differently, the user needs not worry about this issue.

Under the fitted hypothesis (i.e. under the fitted small model) nsim samples of the likelihood ratio test statistic (LRT) are generated.

Then p-values are calculated as follows:

LRT: Assuming that LRT has a chi-square distribution.

PBtest: The fraction of simulated LRT-values that are larger or equal to the observed LRT value.

Bartlett: A Bartlett correction is of LRT is calculated from the mean of the simulated LRT-values

Gamma: The reference distribution of LRT is assumed to be a gamma distribution with mean and variance determined as the sample mean and sample variance of the simulated LRT-values.

F: The LRT divided by the number of degrees of freedom is assumed to be F-distributed, where the denominator degrees of freedom are determined by matching the first moment of the reference distribution.

Note

It can happen that some values of the LRT statistic in the reference distribution are negative. When this happens one will see that the number of used samples (those where the LRT is positive) are reported (this number is smaller than the requested number of samples).

In theory one can not have a negative value of the LRT statistic but in practice on can: We speculate that the reason is as follows: We simulate data under the small model and fit both the small and the large model to the simulated data. Therefore the large model represents - by definition - an over fit; the model has superfluous parameters in it. Therefore the fit of the two models will for some simulated datasets be very similar resulting in similar values of the log-likelihood. There is no guarantee that the the log-likelihood for the large model in practice always will be larger than for the small (convergence problems and other numerical issues can play a role here).

To look further into the problem, one can use the PBrefdist() function for simulating the reference distribution (this reference distribution can be provided as input to PBmodcomp()). Inspection sometimes reveals that while many values are negative, they are numerically very small. In this case one may try to replace the negative values by a small positive value and then invoke PBmodcomp() to get some idea about how strong influence there is on the resulting p-values. (The p-values get smaller this way compared to the case when only the originally positive values are used).

Author(s)

Søren Højsgaard <sorenh@math.aau.dk>

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

See Also

KRmodcomp, PBrefdist

Examples

```
## Not run:
(fm0 <- lmer(Reaction ~ (Days|Subject), sleepstudy))
(fm1 <- lmer(Reaction ~ Days + (Days|Subject), sleepstudy))
```

```
(fm2 <- lmer(Reaction ~ Days + I(Days^2) + (Days|Subject), sleepstudy))

NSIM <- 50 ## Simulations in parametric bootstrap; default is 1000.

## Test for no effect of Days in fm1, i.e. test fm0 under fm1
PBmodcomp(fm1, "Days", cl=1, nsim=NSIM)
PBmodcomp(fm1, ~.-Days, cl=1, nsim=NSIM)
L1 <- cbind(0, 1)
PBmodcomp(fm1, L1, cl=1, nsim=NSIM) ## FIXME
PBmodcomp(fm1, fm0, cl=1, nsim=NSIM)
anova(fm1, fm0)

## Test for no effect of Days and Days-squared in fm2, i.e. test fm0 under fm2
PBmodcomp(fm2, "(Days+I(Days^2))", cl=1, nsim=NSIM)
PBmodcomp(fm2, ~. - Days - I(Days^2), cl=1, nsim=NSIM)
L2 <- rbind(c(0, 1, 0), c(0, 0, 1))
PBmodcomp(fm2, L2, cl=1, nsim=NSIM) ## FIXME
PBmodcomp(fm2, fm0, cl=1, nsim=NSIM)
anova(fm2, fm0)

## Test for no effect of Days-squared in fm2, i.e. test fm1 under fm2
PBmodcomp(fm2, "I(Days^2)", cl=1, nsim=NSIM)
PBmodcomp(fm2, ~. - I(Days^2), cl=1, nsim=NSIM)
L3 <- rbind(c(0, 0, 1))
## PBmodcomp(fm2, L3, cl=1, nsim=NSIM) ## FIXME
PBmodcomp(fm2, fm1, cl=1, nsim=NSIM)
anova(fm2, fm1)

## Linear normal model:
sug <- lm(sugpct ~ block + sow + harvest, data=beets)
sug.h <- update(sug, .~. -harvest)
sug.s <- update(sug, .~. -sow)

PBmodcomp(sug, "harvest", nsim=NSIM, cl=1)
PBmodcomp(sug, ~. - harvest, nsim=NSIM, cl=1)
PBmodcomp(sug, sug.h, nsim=NSIM, cl=1)
anova(sug, sug.h)

## Generalized linear model
mm <- glm(ndead/ntotal ~ sex + log(dose), family=binomial, weight=ntotal, data=budworm)
mm0 <- update(mm, .~. -sex)

### Test for no effect of sex
PBmodcomp(mm, "sex", cl=1, nsim=NSIM)
PBmodcomp(mm, ~.-sex, cl=1, nsim=NSIM)
## PBmodcomp(mm, cbind(0, 1, 0), nsim=NSIM): FIXME
PBmodcomp(mm, mm0, cl=1, nsim=NSIM)
anova(mm, mm0, test="Chisq")

## End(Not run)

## Generalized linear mixed model (it takes a while to fit these)
```

```
## Not run:
(gm1 <- glmer(cbind(incidence, size - incidence) ~ period + (1 | herd),
              data = cbpp, family = binomial))
(gm2 <- update(gm1, .~.-period))

PBmodcomp(gm1, "period", nsim=NSIM)
PBmodcomp(gm1, ~. -period, nsim=NSIM)
PBmodcomp(gm1, gm2, nsim=NSIM)
anova(gm1, gm2)

## End(Not run)


## Not run:
## Linear mixed effects model:
sug   <- lmer(sugpct ~ block + sow + harvest + (1|block:harvest),
              data=beets, REML=FALSE)
sug.h <- update(sug, .~. -harvest)
sug.s <- update(sug, .~. -sow)

anova(sug, sug.h)
PBmodcomp(sug, sug.h, nsim=NSIM, cl=1)
PBmodcomp(sug, "harvest", nsim=NSIM, cl=1)

anova(sug, sug.s)
PBmodcomp(sug, sug.s, nsim=NSIM, cl=1)
PBmodcomp(sug, "sow", nsim=NSIM, cl=1)

## Simulate reference distribution separately:
refdist <- PBrefdist(sug, sug.h, nsim=1000, cl=1)
refdist <- PBrefdist(sug, "harvest", nsim=1000, cl=1)
refdist <- PBrefdist(sug, ~.-harvest, nsim=1000, cl=1)

## Do computations with multiple processors:
## Number of cores:

(nc <- detectCores())
## Create clusters
cl <- makeCluster(rep("localhost", nc))

## Then do:
refdist <- PBrefdist(sug, sug.h, nsim=1000, cl=cl)

## It is recommended to stop the clusters before quitting R:
stopCluster(cl)

## End(Not run)


lm1 <- lm(dist~speed+I(speed^2), data=cars)
PBmodcomp(lm1, .~.-speed, cl=2)
PBmodcomp(lm1, .~.-I(speed^2), cl=2)
```

---

sat__modcomp	*F-test and degrees of freedom based on Satterthwaite approximation*

Description

An approximate F-test based on the Satterthwaite approach.

Usage

```
SATmodcomp(
  largeModel,
  smallModel,
  betaH = 0,
  details = 0,
  eps = sqrt(.Machine$double.eps)
)

## S3 method for class 'lmerMod'
SATmodcomp(
  largeModel,
  smallModel,
  betaH = 0,
  details = 0,
  eps = sqrt(.Machine$double.eps)
)
```

Arguments

largeModel	An `lmer` model
smallModel	An `lmer` model or a restriction matrix
betaH	A number or a vector of the beta of the hypothesis, e.g. L beta=L betaH. If `smallModel` is a model object then betaH=0.
details	If larger than 0 some timing details are printed.
eps	A small number.

Details

Notice: It cannot be guaranteed that the results agree with other implementations of the Satterthwaite approach!

Author(s)

Søren Højsgaard, <sorenh@math.aau.dk>

References

Ulrich Halekoh, Søren Højsgaard (2014)., A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models - The R Package pbkrtest., Journal of Statistical Software, 58(10), 1-30., https://www.jstatsoft.org/v59/i09/

See Also

getKR, lmer, vcovAdj, PBmodcomp, KRmodcomp

Examples

```
(fm0 <- lmer(Reaction ~ (Days|Subject), sleepstudy))
(fm1 <- lmer(Reaction ~ Days + (Days|Subject), sleepstudy))
(fm2 <- lmer(Reaction ~ Days + I(Days^2) + (Days|Subject), sleepstudy))

## Test for no effect of Days in fm1, i.e. test fm0 under fm1
SATmodcomp(fm1, "Days")
SATmodcomp(fm1, ~.-Days)
L1 <- cbind(0, 1)
## SATmodcomp(fm1, L1) ## FIXME
SATmodcomp(fm1, fm0)
anova(fm1, fm0)

## Test for no effect of Days and Days-squared in fm2, i.e. test fm0 under fm2
SATmodcomp(fm2, "(Days+I(Days^2))")
SATmodcomp(fm2, ~. - Days - I(Days^2))
L2 <- rbind(c(0, 1, 0), c(0, 0, 1))
## SATmodcomp(fm2, L2) ## FIXME
SATmodcomp(fm2, fm0)
anova(fm2, fm0)

## Test for no effect of Days-squared in fm2, i.e. test fm1 under fm2
SATmodcomp(fm2, "I(Days^2)")
SATmodcomp(fm2, ~. - I(Days^2))
L3 <- rbind(c(0, 0, 1))
## SATmodcomp(fm2, L3) ## FIXME
SATmodcomp(fm2, fm1)
anova(fm2, fm1)
```

---

x2__modcomp   *Chisq test*

Description

Chisq test

Usage

```
X2modcomp(largeModel, smallModel, betaH = 0, details = 0, ...)

## Default S3 method:
X2modcomp(largeModel, smallModel, betaH = 0, details = 0, ...)
```

Arguments

largeModel	An `lmer` model
smallModel	An `lmer` model or a restriction matrix
betaH	A number or a vector of the beta of the hypothesis, e.g. L beta=L betaH. If smallModel is a model object then betaH=0.
details	If larger than 0 some timing details are printed.
...	Additional arguments, currently not used.

Details

TBW

Author(s)

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(fm0 <- lmer(Reaction ~ (Days|Subject), sleepstudy)) (fm1 <- lmer(Reaction ~ Days + (Days|Subject), sleepstudy)) (fm2 <- lmer(Reaction ~ Days + I(Days^2) + (Days|Subject), sleepstudy))

Test for no effect of Days in fm1, i.e. test fm0 under fm1:
X2modcomp(fm1, "Days") X2modcomp(fm1, ~.-Days) L1 <- cbind(0, 1)

X2modcomp(fm1, L1) ## FIXME:
X2modcomp(fm1, fm0) anova(fm1, fm0)

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