Package 'accessibility'

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Type Package

Title Transport Accessibility Measures

Version 1.4.0

Description A set of fast and convenient functions to help conducting accessibility analyses. Given a pre-computed travel cost matrix and a land use dataset (containing the location of jobs, healthcare and population, for example), the package allows one to calculate accessibility levels and accessibility poverty and inequality. The package covers the majority of the most commonly used accessibility measures (such as cumulative opportunities, gravity-based and floating catchment areas methods), as well as the most frequently used inequality and poverty metrics (such as the Palma ratio, the concentration and Theil indices and the FGT family of measures).

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URL https://github.com/ipeaGIT/accessibility,

https://ipeagit.github.io/accessibility/

BugReports https://github.com/ipeaGIT/accessibility/issues

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balancing_cost

Balancing cost accessibility measure

Description

Calculates the balancing cost measure, which is defined as the travel cost required to reach as many opportunities as the number of people in a given origin. Originally proposed by Barboza et al. (2021), under the name "balancing time".

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
balancing_cost(
  travel_matrix,
  land_use_data,
  opportunity,
  travel_cost,
  demand,
  cost_increment = 1,
  group_by = character(0),
  fill_missing_ids = TRUE
)
```

balancing_cost

Arguments

travel_matrix	A data frame. The travel matrix describing the costs (i.e. travel time, distance, monetary cost, etc.) between the origins and destinations in the study area. Must contain the columns from_id, to_id and any others specified in travel_cost.
land_use_data	A data frame. The distribution of opportunities within the study area cells. Must contain the columns id and any others specified in opportunity.
opportunity	A string. The name of the column in land_use_data with the number of oppor- tunities/resources/services to be considered when calculating accessibility lev- els.
travel_cost	A string. The name of the column in travel_matrix with the travel cost be- tween origins and destinations.
demand	A string. The name of the column in land_use_data with the number of people in each origin that will be considered potential competitors.
cost_increment	A number. The cost increment that should be used when defining the travel cost distribution from which the potential balancing costs will be picked. For example, an increment of 1 tends to suitable for travel time distributions, meaning that the function will first check if any origins reach their balancing cost with a travel time of 0 minutes, then 1 minute, 2 minutes, 3, 4,, etc. A increment of 1 might be too big for a distribution of monetary costs, on the other hand, which could possibly benefit from a smaller increment of 0.05, for example, resulting in the function looking for balancing costs first at a cost of 0, then 0.05, 0.10,, etc. Defaults to 1.
group_by	A character vector. When not character(0) (the default), indicates the travel_matrix columns that should be used to group the accessibility estimates by. For example, if travel_matrix includes a departure time column, that specifies the departure time of each entry in the data frame, passing "departure_time" to this parameter results in accessibility estimates grouped by origin and by departure time.
fill_missing_id	ls
	A logical. When calculating grouped accessibility estimates (i.e. when by_col is not NULL), some combinations of groups and origins may be missing. For example, if a single trip can depart from origin A at 7:15am and reach destination B within 55 minutes, but no trips departing from A at 7:30am can be completed at all, this second combination will not be included in the output. When TRUE (the default), the function identifies which combinations would be left out and fills their respective accessibility values with 0, which incurs in a performance penalty.

Value

A data frame containing the accessibility estimates for each origin/destination (depending if active is TRUE or FALSE) in the travel matrix.

A data frame containing the accessibility estimates for each origin in the travel matrix. Origins marked with a NA balancing cost never reach as many opportunities as there is people residing in them, given the specified travel matrix.

References

Barboza MH, Carneiro MS, Falavigna C, Luz G, Orrico R (2021). "Balancing Time: Using a New Accessibility Measure in Rio de Janeiro." *Journal of Transport Geography*, **90**, 102924. ISSN 09666923, doi:10.1016/j.jtrangeo.2020.102924.

Examples

```
data_dir <- system.file("extdata", package = "accessibility")
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))
bc <- balancing_cost(
   travel_matrix,
   land_use_data,
   opportunity = "jobs",
   travel_cost = "travel_time",
   demand = "population"
)
head(bc)</pre>
```

concentration_index Concentration Index

Description

Calculates the Concentration Index (CI) of a given accessibility distribution. This measures estimates the extent to which accessibility inequalities are systematically associated with individuals' socioeconomic levels. CI values can theoretically vary between -1 and +1 (when all accessibility is concentrated in the most or in the least disadvantaged person, respectively). Negative values indicate that inequalities favor the poor, while positive values indicate a pro-rich bias. The function supports calculating the standard relative CI and the corrected CI, as proposed by Erreygers (2009).

Usage

```
concentration_index(
    accessibility_data,
    sociodemographic_data,
    opportunity,
    population,
    income,
    type,
    group_by = character(0)
)
```

Arguments

accessibility_data		
	A data frame. The accessibility levels whose inequality should be calculated. Must contain the columns id and any others specified in opportunity.	
sociodemograph	ic_data	
	A data frame. The distribution of sociodemographic characteristics of the pop- ulation in the study area cells. Must contain the columns id and any others specified in population and income.	
opportunity	A string. The name of the column in accessibility_data with the accessibil- ity levels to be considerend when calculating inequality levels.	
population	A string. The name of the column in sociodemographic_data with the num- ber of people in each cell. Used to weigh accessibility levels when calculating inequality.	
income	A string. The name of the column in sociodemographic_data with the income variable that should be used to sort the population from the least to the most privileged. Please note that this variable should describe income per capita (e.g. mean income per capita, household income per capita, etc), instead of the to- tal amount of income in each cell. Also note that, while income is generally used to rank population groups, any variable that can be used to describe one's socioeconomic status, such as education level, can be passed to this argument, as long as it can be numerically ordered (in which higher values denote higher socioeconomic status).	
type	A string. Which type of Concentration Index to calculate. Current available options are "standard" and "corrected".	
group_by	A character vector. When not character(0) (the default), indicates the accessibility_data columns that should be used to group the inequality estimates by. For example, if accessibility_data includes a scenario column that identifies distinct scenarios that each accessibility estimates refer to (e.g. before and after a transport policy intervention), passing "scenario" to this parameter results in inequality estimates grouped by scenario.	

Value

A data frame containing the inequality estimates for the study area.

References

Erreygers G (2009). "Correcting the Concentration Index." *Journal of Health Economics*, **28**(2), 504–515. ISSN 0167-6296, doi:10.1016/j.jhealeco.2008.02.003.

See Also

Other inequality: gini_index(), palma_ratio(), theil_t()

Examples

```
data_dir <- system.file("extdata", package = "accessibility")</pre>
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))</pre>
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))</pre>
access <- cumulative_cutoff(</pre>
  travel_matrix,
  land_use_data,
  cutoff = 30,
  opportunity = "jobs",
  travel_cost = "travel_time"
)
ci <- concentration_index(</pre>
  access,
  sociodemographic_data = land_use_data,
  opportunity = "jobs",
  population = "population",
  income = "income_per_capita",
  type = "corrected"
)
ci
```

cost_to_closest Minimum travel cost to closest N number of opportunities

Description

Calculates the minimum travel cost to the closest N number of opportunities.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
cost_to_closest(
  travel_matrix,
  land_use_data,
  opportunity,
  travel_cost,
  n = 1,
  group_by = character(0),
  active = TRUE,
  fill_missing_ids = TRUE
)
```

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Arguments

travel_matrix	A data frame. The travel matrix describing the costs (i.e. travel time, distance, monetary cost, etc.) between the origins and destinations in the study area. Must contain the columns from_id, to_id and any others specified in travel_cost.
land_use_data	A data frame. The distribution of opportunities within the study area cells. Must contain the columns id and any others specified in opportunity.
opportunity	A string. The name of the column in land_use_data with the number of oppor- tunities/resources/services to be considered when calculating accessibility lev- els.
travel_cost	A string. The name of the column in travel_matrix with the travel cost be- tween origins and destinations.
n	A numeric vector. The minimum number of opportunities that should be con- sidered. Defaults to 1. If more than one value is provided, the output includes an extra column specifying the number of opportunities that the minimum travel cost refers to.
group_by	A character vector. When not character(0) (the default), indicates the travel_matrix columns that should be used to group the accessibility estimates by. For example, if travel_matrix includes a departure time column, that specifies the departure time of each entry in the data frame, passing "departure_time" to this parameter results in accessibility estimates grouped by origin and by departure time.
active	A logical. Whether to calculate active accessibility (the of opportunities that can be reached from a given origin, the default) or passive accessibility (by how many people each destination can be reached).
fill_missing_id	ls
	A logical. Calculating minimum travel cost to closest N number of opportu- nities may result in missing ids in the output if they cannot reach the specified amount of opportunities across all destinations they can reach. For example, estimating the minimum travel time that an origin that can only reach 4 opportu- nities takes to reach 5 opportunities resulting in such origin not being included in the output. When TRUE (the default), the function identifies which ids would

Value

A data frame containing the accessibility estimates for each origin/destination (depending if active is TRUE or FALSE) in the travel matrix.

be left out from the output and fill their respective minimum travel costs with

Examples

```
data_dir <- system.file("extdata", package = "accessibility")
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))</pre>
```

Inf, which incurs in a performance penalty.

df <- cost_to_closest(

```
travel_matrix,
land_use_data,
n = 1,
opportunity = "schools",
travel_cost = "travel_time"
)
head(df)
df <- cost_to_closest(
travel_matrix,
land_use_data,
n = c(1, 2),
opportunity = "schools",
travel_cost = "travel_time"
)
head(df)
```

cumulative_cutoff Cumulative access based on a travel cost cutoff

Description

Calculates the number of opportunities accessible under a given specified travel cost cutoff.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
cumulative_cutoff(
  travel_matrix,
  land_use_data,
  opportunity,
  travel_cost,
  cutoff,
  group_by = character(0),
  active = TRUE,
  fill_missing_ids = TRUE
)
```

Arguments

travel_matrix	A data frame. The travel matrix describing the costs (i.e. travel time, distance,
	monetary cost, etc.) between the origins and destinations in the study area. Must
	contain the columns from_id, to_id and any others specified in travel_cost.
land_use_data	A data frame. The distribution of opportunities within the study area cells. Must contain the columns id and any others specified in opportunity.

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- opportunity A string. The name of the column in land_use_data with the number of opportunities/resources/services to be considered when calculating accessibility levels.
- travel_cost A character vector. The name of the columns in travel_matrix with the travel costs between origins and destinations to be considered in the calculation.
- cutoff Either a numeric vector or a list of numeric vectors, one for each cost specified in travel_cost. The travel cost cutoffs to consider when calculating accessibility levels. If a list, the function finds every single possible cutoff combination and use them to calculate accessibility (e.g. if one specifies that travel time cutoffs should be 30 and 60 minutes and that monetary cost cutoffs should be 5 and 10 dollars, the output includes accessibility estimates limited at 30 min & 5 dollars, 30 min & 10 dollars, 60 min & 5 dollars and 60 min & 10 dollars). In these cases, cost constraints are considered simultaneously - i.e. only trips that take 30 minutes or less AND 5 dollars or less to be completed, for example, are included in the accessibility output. The cutoff parameter is not included in the final output if the input includes only a single cutoff for a single travel cost.
- group_by A character vector. When not character(0) (the default), indicates the travel_matrix columns that should be used to group the accessibility estimates by. For example, if travel_matrix includes a departure time column, that specifies the departure time of each entry in the data frame, passing "departure_time" to this parameter results in accessibility estimates grouped by origin and by departure time.
- active A logical. Whether to calculate active accessibility (the of opportunities that can be reached from a given origin, the default) or passive accessibility (by how many people each destination can be reached).

fill_missing_ids

A logical. Calculating cumulative accessibility may result in missing ids if the they cannot reach any of the destinations within the specified travel cost cutoff. For example, using a travel time cutoff of 20 minutes, when estimating the accessibility of origin A that can only reach destinations with more than 40 minutes results in id A not being included in the output. When TRUE (the default), the function identifies which origins would be left out and fills their respective accessibility values with 0, which incurs in a performance penalty.

Value

A data frame containing the accessibility estimates for each origin/destination (depending if active is TRUE or FALSE) in the travel matrix.

See Also

Other cumulative access: cumulative_interval()

```
data_dir <- system.file("extdata", package = "accessibility")
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))</pre>
```

```
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))</pre>
# active accessibility: number of schools accessible from each origin
df <- cumulative_cutoff(</pre>
  travel_matrix = travel_matrix,
  land_use_data = land_use_data,
  cutoff = 30,
  opportunity = "schools",
  travel_cost = "travel_time"
)
head(df)
df <- cumulative_cutoff(</pre>
  travel_matrix = travel_matrix,
  land_use_data = land_use_data,
  cutoff = c(30, 60),
  opportunity = "schools",
  travel_cost = "travel_time"
)
head(df)
# passive accessibility: number of people that can reach each destination
df <- cumulative_cutoff(</pre>
  travel_matrix = travel_matrix,
  land_use_data = land_use_data,
  cutoff = 30,
  opportunity = "population",
  travel_cost = "travel_time",
  active = FALSE
)
head(df)
# using multiple travel costs
pareto_frontier <- readRDS(file.path(data_dir, "pareto_frontier.rds"))</pre>
df <- cumulative_cutoff(</pre>
  pareto_frontier,
  land_use_data = land_use_data,
  opportunity = "jobs",
  travel_cost = c("travel_time", "monetary_cost"),
  cutoff = list(c(20, 30), c(0, 5, 10))
)
head(df)
```

cumulative_interval Cumulative access based on maximum travel time interval

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Description

Calculates the average or median number of opportunities that can be reached considering multiple maximum travel cost thresholds within a given travel cost interval specified by the user. The time interval cumulative accessibility measures was originally proposed by Tomasiello et al. (2023).

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
cumulative_interval(
  travel_matrix,
  land_use_data,
  opportunity,
  travel_cost,
  interval,
  interval_increment = 1,
  summary_function = stats::median,
  group_by = character(0),
  active = TRUE
)
```

Arguments

travel_matrix	A data frame. The travel matrix describing the costs (i.e. travel time, distance, monetary cost, etc.) between the origins and destinations in the study area. Must contain the columns from_id, to_id and any others specified in travel_cost.	
land_use_data	A data frame. The distribution of opportunities within the study area cells. Must contain the columns id and any others specified in opportunity.	
opportunity	A string. The name of the column in land_use_data with the number of oppor- tunities/resources/services to be considered when calculating accessibility lev- els.	
travel_cost	A string. The name of the column in travel_matrix with the travel cost between origins and destinations.	
interval	A numeric vector of length 2. Indicates the start and end points of the interval of travel cost thresholds to be used. The first entry must be lower than the second.	
interval_increment		
	A numeric. How many travel cost units separate the cutoffs used to calculate the accessibility estimates which will be used to calculate the summary estimate within the specified interval. Should be thought as the resolution of the distribution of travel costs within the interval. Defaults to 1.	
summary_function		
	A function. This function is used to summarize a distribution of accessibility estimates within a travel cost interval as a single value. Can be any function that takes an arbitrary number of numeric values as as input and returns a single	

number as output. Defaults to stats::median().

group_by	A character vector. When not character(0) (the default), indicates the travel_matrix columns that should be used to group the accessibility estimates by. For example, if travel_matrix includes a departure time column, that specifies the departure time of each entry in the data frame, passing "departure_time" to this parameter results in accessibility estimates grouped by origin and by departure time.
active	A logical. Whether to calculate active accessibility (the of opportunities that can be reached from a given origin, the default) or passive accessibility (by how many people each destination can be reached).

Value

A data frame containing the accessibility estimates for each origin/destination (depending if active is TRUE or FALSE) in the travel matrix.

References

Tomasiello DB, Herszenhut D, Oliveira JLA, Braga CKV, Pereira RHM (2023). "A Time Interval Metric for Cumulative Opportunity Accessibility." *Applied Geography*, **157**, 103007. ISSN 0143-6228, doi:10.1016/j.apgeog.2023.103007.

See Also

Other cumulative access: cumulative_cutoff()

```
data_dir <- system.file("extdata", package = "accessibility")</pre>
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))</pre>
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))</pre>
df <- cumulative_interval(</pre>
  travel_matrix = travel_matrix,
  land_use_data = land_use_data,
  interval = c(20, 30),
  opportunity = "schools",
  travel_cost = "travel_time"
)
head(df)
df <- cumulative_interval(</pre>
  travel_matrix = travel_matrix,
  land_use_data = land_use_data,
  interval = c(40, 80),
  opportunity = "jobs",
  travel_cost = "travel_time"
)
head(df)
```

decay_binary

Description

Returns a binary weighting function (frequently used to calculate cumulative opportunities measures) to be used inside accessibility calculating functions.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
decay_binary(cutoff)
```

Arguments

cutoff A numeric vector. The numbers indicating the travel cost cutoffs.

Value

A function that takes a generic travel cost vector (numeric) as an input and returns a list of weight vectors (a list of numeric vectors, named after the arguments passed to the decay function).

See Also

Other decay functions: decay_exponential(), decay_linear(), decay_logistic(), decay_power(),
decay_stepped()

```
weighting_function <- decay_binary(cutoff = 30)
weighting_function(c(20, 35))
weighting_function <- decay_binary(cutoff = c(30, 45))
weighting_function(c(20, 35))</pre>
```

decay_exponential Negative exponential decay function

Description

Returns a negative exponential weighting function to be used inside accessibility calculating functions.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
decay_exponential(decay_value)
```

Arguments

decay_value A numeric vector. The calibration parameters that, when multiplied by the travel cost, are used as the exponent of e in the negative exponential function.

Value

A function that takes a generic travel cost vector (numeric) as an input and returns a list of weight vectors (a list of numeric vectors, named after the arguments passed to the decay function).

See Also

Other decay functions: decay_binary(), decay_linear(), decay_logistic(), decay_power(),
decay_stepped()

```
weighting_function <- decay_exponential(decay_value = 0.1)
weighting_function(c(20, 30))
weighting_function <- decay_exponential(decay_value = c(0.1, 0.2))
weighting_function(c(20, 30))</pre>
```

decay_linear

Description

Returns a linear weighting function to be used inside accessibility calculating functions.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

decay_linear(cutoff)

Arguments

cutoff	A numeric vector. Indicates the travel cost cutoffs until which the weighting
	factor decays linearly. From this point onward the weight is equal to 0.

Value

A function that takes a generic travel cost vector (numeric) as an input and returns a list of weight vectors (a list of numeric vectors, named after the arguments passed to the decay function).

See Also

Other decay functions: decay_binary(), decay_exponential(), decay_logistic(), decay_power(),
decay_stepped()

```
weighting_function <- decay_linear(cutoff = 30)
weighting_function(c(20, 35))
weighting_function <- decay_linear(cutoff = c(30, 45))
weighting_function(c(20, 35))</pre>
```

decay_logistic

Description

Returns a logistic weighting function (in which the weights follow the distribution of a reversed cumulative logistic curve) to be used inside accessibility calculating functions. The logistic curve is parameterized with the cutoff that sets its inflection point and the standard deviation that sets its steepness.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

decay_logistic(cutoff, sd)

Arguments

cutoff	A numeric vector. The cost value that serves as the inflection point of the cumulative logistic curve.
sd	A numeric vector with same length as cutoff. The standard deviation of the logistic curve. Values near 0 result in weighting curves that approximate binary decay, while higher values tend to linearize the decay.

Details

When using a function created with decay_logistic(), the output is named after the combination of cutoffs ("c") and standard deviations ("sd") - e.g. given the cutoffs c(30, 40) and the standard deviations c(10, 20), the first element of the output will be named "c30; sd10" and the second will be named "c40; sd20". This function uses the adjusted logistic decay curve proposed by Bauer and Groneberg (2016), in which the condition f(0) = 1 is met (i.e. the weight of an opportunity whose cost to reach is 0 is 1).

Value

A function that takes a generic travel cost vector (numeric) as input and returns a vector of weights (numeric).

References

Bauer J, Groneberg DA (2016). "Measuring Spatial Accessibility of Health Care Providers – Introduction of a Variable Distance Decay Function within the Floating Catchment Area (FCA) Method." *PLOS ONE*, **11**(7), e0159148. ISSN 1932-6203, doi:10.1371/journal.pone.0159148.

See Also

Other decay functions: decay_binary(), decay_exponential(), decay_linear(), decay_power(),
decay_stepped()

decay_power

Examples

```
weighting_function <- decay_logistic(cutoff = 30, sd = 5)
weighting_function(c(0, 30, 45, 60))
weighting_function <- decay_logistic(cutoff = c(30, 45), sd = c(5, 10))
weighting_function(c(0, 30, 45, 60))</pre>
```

decay_power Inverse power decay function

Description

Returns an inverse power weighting function to be used inside accessibility calculating functions. This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
decay_power(decay_value)
```

Arguments

decay_value A numeric vector. The calibration parameters to be used as the exponents in the inverse power function.

Value

A function that takes a generic travel cost vector (numeric) as an input and returns a list of weight vectors (a list of numeric vectors, named after the arguments passed to the decay function).

See Also

Other decay functions: decay_binary(), decay_exponential(), decay_linear(), decay_logistic(), decay_stepped()

```
weighting_function <- decay_power(decay_value = 0.1)
weighting_function(c(20, 35))
weighting_function <- decay_power(decay_value = c(0.1, 0.2))
weighting_function(c(20, 35))</pre>
```

decay_stepped

Description

Returns a stepped weighting function to be used inside accessibility calculating functions.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

decay_stepped(steps, weights)

Arguments

steps	A numeric vector or a list of numeric vectors. The travel cost steps, in ascending order. Please do not include travel cost 0 as a step: this is already handled by the function.
weights	A numeric vector with same length as steps or a list of numeric vectors whose lengths are equal to the lengths of the elements of same index in steps. The values, between 0 and 1, that the function assumes at each step. Please do not include weight 1 as the first value: this is already handled by the function. The function considers the steps' intervals "open on the right", meaning that the function assumes the step value at the actual step, not right after it. Please see the illustrative examples for effects of this assumption on the results.

Details

When both steps and weights parameters are given lists, their content are matched element-wise to define each stepped weighting function

- i.e. the first element of steps is matched to the first element of weights, the second element of steps is matched to the second of weights, etc. When using a function created with decay_stepped(), the output is named after the combination of steps ("s") and weights ("w")
- e.g. given the steps c(10, 20, 30) and the weights c(0.66, 0.33, 0), the output will be named "s(10, 20, 30);w(0.66, 0.33, 0)".

Value

A function that takes a generic travel cost vector (numeric) as an input and returns a vector of weights (numeric).

See Also

Other decay functions: decay_binary(), decay_exponential(), decay_linear(), decay_logistic(), decay_power()

fgt_poverty

Examples

```
weighting_function <- decay_stepped(
    c(10, 20, 30, 40),
    weights = c(0.75, 0.5, 0.25, 0)
)
weighting_function(c(5, 25, 35, 45))
weighting_function <- decay_stepped(
    list(c(10, 20, 30, 40), c(10, 20, 30, 40)),
    weights = list(c(0.75, 0.5, 0.25, 0), c(0.8, 0.6, 0.4, 0.2))
)
weighting_function(c(5, 25, 35, 45))
# intervals are open on the right, so the values change exactly at each step
weighting_function(c(0, 10, 20, 30, 40))
```

fgt_poverty

Foster-Greer-Thorbecke (FGT) poverty measures

Description

Calculates the FGT metrics, a family of poverty measures originally proposed by Foster et al. (1984) that capture the extent and severity of poverty within an accessibility distribution. The FGT family is composed of three measures that differ based on the α parameter used to calculate them (either 0, 1 or 2) and which also changes their interpretation. Please see the details section for more information on the interpretation of the measures.

Usage

```
fgt_poverty(
    accessibility_data,
    sociodemographic_data,
    opportunity,
    population,
    poverty_line,
    group_by = character(0)
)
```

Arguments

```
accessibility_data
```

A data frame. The accessibility levels whose poverty levels should be calculated. Must contain the columns id and any others specified in opportunity.

sociodemographic_data		
	A data frame. The distribution of sociodemographic characteristics of the pop- ulation in the study area cells. Must contain the columns id and any others specified in population.	
opportunity	A string. The name of the column in accessibility_data with the accessibil- ity levels to be considerend when calculating accessibility poverty.	
population	A string. The name of the column in sociodemographic_data with the num- ber of people in each cell. Used to weigh accessibility levels when calculating poverty.	
poverty_line	A numeric. The poverty line below which individuals are considered to be in accessibility poverty.	
group_by	A character vector. When not character(0) (the default), indicates the accessibility_data columns that should be used to group the poverty estimates by. For example, if accessibility_data includes a race column that specifies the racial category of the population (e.g. "black" and "white") that each entry refers to, passing "race" to this parameter results in poverty estimates grouped by race.	

Value

A data frame containing the three poverty estimates (FGT0, FGT1 and FGT2) for the study area.

Interpretation of FGT measures

The interpretation of each FGT measure depends on the α parameter used to calculate it:

- with $\alpha = 0$ (FGT0) the measure captures the *extent* of poverty as a simple headcount i.e. the proportion of people below the poverty line;
- with $\alpha = 1$ (FGT1) the measure, also know as the "poverty gap index", captures the *severity* of poverty as the average percentage distance between the poverty line and the accessibility of individuals below the poverty line;
- with $\alpha = 2$ (FGT2) the measure simultaneously captures the *extent* and the *severity* of poverty by calculating the number of people below the poverty line weighted by the size of the accessibility shortfall relative to the poverty line.

FGT values range from 0 to 1. A value of 0 indicates that every individual is above the poverty line. When every individual is below the poverty line, however, FGT0 value is 1 and FGT1 and FGT2 values approach 1.

References

Foster J, Greer J, Thorbecke E (1984). "A Class of Decomposable Poverty Measures." *Econometrica*, **52**(3), 761–766. ISSN 0012-9682, doi:10.2307/1913475, 1913475.

```
data_dir <- system.file("extdata", package = "accessibility")
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))</pre>
```

```
access <- cumulative_cutoff(
  travel_matrix,
  land_use_data,
  cutoff = 30,
  opportunity = "jobs",
  travel_cost = "travel_time"
)
poverty <- fgt_poverty(
  access,
  opportunity = "jobs",
  sociodemographic_data = land_use_data,
  population = "population",
  poverty_line = 95368
)
poverty</pre>
```

floating_catchment_area

Floating catchment area accessibility

Description

Calculates accessibility accounting for the competition of resources using a measure from the floating catchment area (FCA) family. Please see the details for the available FCA measures.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
floating_catchment_area(
   travel_matrix,
   land_use_data,
   opportunity,
   travel_cost,
   demand,
   method,
   decay_function,
   group_by = character(0),
   fill_missing_ids = TRUE
)
```

Arguments

travel_matrix A data frame. The travel matrix describing the costs (i.e. travel time, distance, monetary cost, etc.) between the origins and destinations in the study area. Must contain the columns from_id, to_id and any others specified in travel_cost.

land_use_data	A data frame. The distribution of opportunities within the study area cells. Must contain the columns id and any others specified in opportunity.
opportunity	A string. The name of the column in land_use_data with the number of oppor- tunities/resources/services to be considered when calculating accessibility lev- els.
travel_cost	A string. The name of the column in travel_matrix with the travel cost be- tween origins and destinations.
demand	A string. The name of the column in land_use_data with the number of people in each origin that will be considered potential competitors.
method	A string. Which floating catchment area measure to use. Current available op- tions are "2sfca" and "bfca". More info in the details.
decay_function	A fuction that converts travel cost into an impedance factor used to weight opportunities. This function should take a numeric vector and also return a numeric vector as output, with the same length as the input. For convenience, the package currently includes the following functions: decay_binary(), decay_exponential(), decay_power() and decay_stepped(). See the documentation of each decay function for more details.
group_by	A character vector. When not character(0) (the default), indicates the travel_matrix columns that should be used to group the accessibility estimates by. For example, if travel_matrix includes a departure time column, that specifies the departure time of each entry in the data frame, passing "departure_time" to this parameter results in accessibility estimates grouped by origin and by departure time.
fill_missing_id	
	A logical. When calculating grouped accessibility estimates (i.e. when by_col is not NULL), some combinations of groups and origins may be missing. For ex- ample, if a single trip can depart from origin A at 7:15am and reach destination B within 55 minutes, but no trips departing from A at 7:30am can be completed at all, this second combination will not be included in the output. When TRUE (the default), the function identifies which combinations would be left out and fills their respective accessibility values with 0, which incurs in a performance

Value

A data frame containing the accessibility estimates for each origin/destination (depending if active is TRUE or FALSE) in the travel matrix.

Details

The package currently includes two built-in FCA measures:

penalty.

- 2SFCA the 2-Step Floating Catchment Area measure was the first accessibility metric in the FCA family. It was originally proposed by Luo and Wang (2003).
- BFCA the Balanced Floating Catchment Area measure calculates accessibility accounting for competition effects while simultaneously correcting for issues of inflation of demand and service levels that are present in other FCA measures. It was originally proposed by Paez et al. (2019) and named in Pereira et al. (2021).

gini_index

References

Luo W, Wang F (2003). "Measures of Spatial Accessibility to Health Care in a GIS Environment: Synthesis and a Case Study in the Chicago Region." *Environment and Planning B: Planning and Design*, **30**(6), 865–884. ISSN 0265-8135, 1472-3417, doi:10.1068/b29120.

Paez A, Higgins CD, Vivona SF (2019). "Demand and Level of Service Inflation in Floating Catchment Area (FCA) Methods." *PLOS ONE*, **14**(6), e0218773. ISSN 1932-6203, doi:10.1371/journal.pone.0218773.

Pereira RHM, Braga CKV, Servo LM, Serra B, Amaral P, Gouveia N, Paez A (2021). "Geographic Access to COVID-19 Healthcare in Brazil Using a Balanced Float Catchment Area Approach." *Social Science & Medicine*, **273**, 113773. ISSN 0277-9536, doi:10.1016/j.socscimed.2021.113773.

Examples

```
data_dir <- system.file("extdata", package = "accessibility")</pre>
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))</pre>
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))</pre>
# 2SFCA with a step decay function
df <- floating_catchment_area(</pre>
 travel_matrix,
 land_use_data,
 method = "2sfca",
 decay_function = decay_binary(cutoff = 50),
 opportunity = "jobs",
 travel_cost = "travel_time",
 demand = "population"
)
head(df)
# BFCA with an exponential decay function
df <- floating_catchment_area(</pre>
 travel_matrix,
 land_use_data,
 method = "bfca",
 decay_function = decay_exponential(decay_value = 0.5),
 opportunity = "jobs",
 travel_cost = "travel_time",
 demand = "population"
)
head(df)
```

gini_index

Gini Index

Description

Calculates the Gini Index of a given accessibility distribution.

Usage

```
gini_index(
    accessibility_data,
    sociodemographic_data,
    opportunity,
    population,
    group_by = character(0)
)
```

Arguments

accessibility_data

A data frame. The accessibility levels whose inequality should be calculated. Must contain the columns id and any others specified in opportunity.

sociodemographi	.c_data
	A data frame. The distribution of sociodemographic characteristics of the pop- ulation in the study area cells. Must contain the columns id and any others specified in population.
opportunity	A string. The name of the column in accessibility_data with the accessibil- ity levels to be considerend when calculating inequality levels.
population	A string. The name of the column in sociodemographic_data with the num- ber of people in each cell. Used to weigh accessibility levels when calculating inequality.
group_by	A character vector. When not character(0) (the default), indicates the accessibility_data columns that should be used to group the inequality estimates by. For example, if accessibility_data includes a scenario column that identifies distinct scenarios that each accessibility estimates refer to (e.g. before and after a transport policy intervention), passing "scenario" to this parameter results in inequality estimates grouped by scenario.

Value

A data frame containing the inequality estimates for the study area.

See Also

Other inequality: concentration_index(), palma_ratio(), theil_t()

Examples

```
data_dir <- system.file("extdata", package = "accessibility")
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))</pre>
```

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gravity

```
access <- cumulative_cutoff(
  travel_matrix,
  land_use_data,
  cutoff = 30,
  opportunity = "jobs",
  travel_cost = "travel_time"
)
gini <- gini_index(
  access,
  sociodemographic_data = land_use_data,
  opportunity = "jobs",
  population = "population"
)
gini</pre>
```

```
gravity
```

Gravity-based accessibility measures

Description

Calculates gravity-based accessibility using a decay function specified by the user.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
gravity(
  travel_matrix,
  land_use_data,
  opportunity,
  travel_cost,
  decay_function,
  group_by = character(0),
  active = TRUE,
  fill_missing_ids = TRUE
)
```

Arguments

travel_matrix	A data frame. The travel matrix describing the costs (i.e. travel time, distance,
	monetary cost, etc.) between the origins and destinations in the study area. Must
	contain the columns from_id, to_id and any others specified in travel_cost.
land_use_data	A data frame. The distribution of opportunities within the study area cells. Must contain the columns id and any others specified in opportunity.

opportunity	A string. The name of the column in land_use_data with the number of oppor- tunities/resources/services to be considered when calculating accessibility lev- els.
travel_cost	A string. The name of the column in travel_matrix with the travel cost be- tween origins and destinations.
decay_function	A fuction that converts travel cost into an impedance factor used to weight opportunities. This function should take a numeric vector and also return a numeric vector as output, with the same length as the input. For convenience, the package currently includes the following functions: decay_binary(), decay_exponential(), decay_power() and decay_stepped(). See the documentation of each decay function for more details.
group_by	A character vector. When not character(0) (the default), indicates the travel_matrix columns that should be used to group the accessibility estimates by. For example, if travel_matrix includes a departure time column, that specifies the departure time of each entry in the data frame, passing "departure_time" to this parameter results in accessibility estimates grouped by origin and by departure time.
active	A logical. Whether to calculate active accessibility (the of opportunities that can be reached from a given origin, the default) or passive accessibility (by how many people each destination can be reached).
fill_missing_id	S
	A logical. When calculating grouped accessibility estimates (i.e. when by_col is not NULL), some combinations of groups and origins may be missing. For example, if a single trip can depart from origin A at 7:15am and reach destination B within 55 minutes, but no trips departing from A at 7:30am can be completed at all, this second combination will not be included in the output. When TRUE (the default), the function identifies which combinations would be left out and fills their respective accessibility values with 0, which incurs in a performance penalty.

Value

A data frame containing the accessibility estimates for each origin/destination (depending if active is TRUE or FALSE) in the travel matrix.

```
data_dir <- system.file("extdata", package = "accessibility")
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))
df_linear <- gravity(
   travel_matrix,
   land_use_data,
   decay_function = decay_linear(cutoff = 50),
   opportunity = "schools",
   travel_cost = "travel_time"
)</pre>
```

palma_ratio

```
head(df_linear)

df_exp <- gravity(
  travel_matrix,
  land_use_data,
  decay_function = decay_exponential(decay_value = 0.5),
  opportunity = "schools",
  travel_cost = "travel_time"
)
head(df_exp)</pre>
```

palma_ratio

Palma Ratio

Description

Calculates the Palma Ratio of a given accessibility distribution. Originally defined as the income share of the richest 10% of a population divided by the income share of the poorest 40%, this measure has been adapted in transport planning as the average accessibility of the richest 10% divided by the average accessibility of the poorest 40%.

Usage

```
palma_ratio(
    accessibility_data,
    sociodemographic_data,
    opportunity,
    population,
    income,
    group_by = character(0)
)
```

Arguments

accessibility_d	lata
	A data frame. The accessibility levels whose inequality should be calculated.
	Must contain the columns id and any others specified in opportunity.
sociodemographic_data	
	A data frame. The distribution of sociodemographic characteristics of the population in the study area cells. Must contain the columns id and any others specified in population and income.
opportunity	A string. The name of the column in accessibility_data with the accessibility levels to be considerend when calculating inequality levels.
population	A string. The name of the column in sociodemographic_data with the number of people in each cell. Used to weigh accessibility levels when calculating inequality.

income	A string. The name of the column in sociodemographic_data with the income variable that should be used to classify the population in socioeconomic groups. Please note that this variable should describe income per capita (e.g. mean income per capita, household income per capita, etc), instead of the total amount of income in each cell.
group_by	A character vector. When not character(0) (the default), indicates the accessibility_data columns that should be used to group the inequality estimates by. For example, if accessibility_data includes a scenario column that identifies distinct scenarios that each accessibility estimates refer to (e.g. before and after a transport policy intervention), passing "scenario" to this parameter results in inequality estimates grouped by scenario.

Value

A data frame containing the inequality estimates for the study area.

See Also

Other inequality: concentration_index(), gini_index(), theil_t()

```
data_dir <- system.file("extdata", package = "accessibility")</pre>
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))</pre>
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))</pre>
access <- cumulative_cutoff(</pre>
  travel_matrix,
  land_use_data,
  cutoff = 30,
  opportunity = "jobs",
  travel_cost = "travel_time"
)
palma <- palma_ratio(</pre>
  access,
  sociodemographic_data = land_use_data,
  opportunity = "jobs",
  population = "population",
  income = "income_per_capita"
)
palma
```

spatial_availability Spatial availability

Description

Calculates spatial availability, an accessibility measured proposed by Soukhov et al. (2023) that takes into account competition effects. The accessibility levels that result from using this measure are proportional both to the demand in each origin and to the travel cost it takes to reach the destinations.

This function is generic over any kind of numeric travel cost, such as distance, time and money.

Usage

```
spatial_availability(
  travel_matrix,
  land_use_data,
  opportunity,
  travel_cost,
  demand,
  decay_function,
  alpha = 1,
  group_by = character(0),
  fill_missing_ids = TRUE,
  detailed_results = FALSE
)
```

Arguments

travel_matrix	A data frame. The travel matrix describing the costs (i.e. travel time, distance, monetary cost, etc.) between the origins and destinations in the study area. Must contain the columns from_id, to_id and any others specified in travel_cost.
land_use_data	A data frame. The distribution of opportunities within the study area cells. Must contain the columns id and any others specified in opportunity.
opportunity	A string. The name of the column in land_use_data with the number of oppor- tunities/resources/services to be considered when calculating accessibility lev- els.
travel_cost	A string. The name of the column in travel_matrix with the travel cost be- tween origins and destinations.
demand	A string. The name of the column in land_use_data with the number of people in each origin that will be considered potential competitors.
decay_function	A fuction that converts travel cost into an impedance factor used to weight opportunities. This function should take a numeric vector and also return a numeric vector as output, with the same length as the input. For convenience, the package currently includes the following functions: decay_binary(), decay_exponential(), decay_power() and decay_stepped(). See the documentation of each decay function for more details.

alpha	A numeric. A parameter used to modulate the effect of demand by population. When less than 1, opportunities are allocated more rapidly to smaller centers relative to larger ones; values higher than 1 achieve the opposite effect.
group_by	A character vector. When not character(\emptyset) (the default), indicates the travel_matrix columns that should be used to group the accessibility estimates by. For example, if travel_matrix includes a departure time column, that specifies the departure time of each entry in the data frame, passing "departure_time" to this parameter results in accessibility estimates grouped by origin and by departure time.
fill_missing_ids	
	A logical. When calculating grouped accessibility estimates (i.e. when by_col is not NULL), some combinations of groups and origins may be missing. For example, if a single trip can depart from origin A at 7:15am and reach destination B within 55 minutes, but no trips departing from A at 7:30am can be completed at all, this second combination will not be included in the output. When TRUE (the default), the function identifies which combinations would be left out and fills their respective accessibility values with 0, which incurs in a performance penalty.
detailed_result	S
	A logical. Whether to return spatial availability results aggregated by origin-

A logical. Whether to return spatial availability results aggregated by origindestination pair (TRUE) or by origin (FALSE, the default). When TRUE, the output also includes the demand, impedance and combined balancing factors used to calculate spatial availability. Please note that the argument fill_missing_ids does not affect the output when detailed_results is TRUE.

Value

A data frame containing the accessibility estimates for each origin/destination (depending if active is TRUE or FALSE) in the travel matrix.

References

Soukhov A, Páez A, Higgins CD, Mohamed M (2023). "Introducing Spatial Availability, a Singly-Constrained Measure of Competitive Accessibility." *PLOS ONE*, **18**(1), e0278468. ISSN 1932-6203, doi:10.1371/journal.pone.0278468.

```
# the example below is based on Soukhov et al. (2023) paper
```

```
travel_matrix <- data.table::data.table(
  from_id = rep(c("A", "B", "C"), each = 3),
  to_id = as.character(rep(1:3, 3)),
  travel_time = c(15, 30, 100, 30, 15, 100, 100, 100, 15)
)
land_use_data <- data.table::data.table(
  id = c("A", "B", "C", "1", "2", "3"),
  population = c(50000, 150000, 10000, 0, 0, 0),
  jobs = c(0, 0, 0, 100000, 100000, 10000)</pre>
```

theil_t

```
)
df <- spatial_availability(</pre>
  travel_matrix,
  land_use_data,
  opportunity = "jobs",
  travel_cost = "travel_time",
  demand = "population",
  decay_function = decay_exponential(decay_value = 0.1)
)
df
detailed_df <- spatial_availability(</pre>
  travel_matrix,
  land_use_data,
  opportunity = "jobs",
  travel_cost = "travel_time",
  demand = "population",
  decay_function = decay_exponential(decay_value = 0.1),
  detailed_results = TRUE
)
detailed_df
```

theil_t

Theil T Index

Description

Calculates the Theil T Index of a given accessibility distribution. Values range from 0 (when all individuals have exactly the same accessibility levels) to the natural log of n, in which n is the number of individuals in the accessibility dataset. If the individuals can be classified into mutually exclusive and completely exhaustive groups, the index can be decomposed into a between-groups inequally component and a within-groups component.

Usage

```
theil_t(
    accessibility_data,
    sociodemographic_data,
    opportunity,
    population,
    socioeconomic_groups = NULL,
    group_by = character(0)
)
```

Arguments

accessibility_data	
	A data frame. The accessibility levels whose inequality should be calculated. Must contain the columns id and any others specified in opportunity.
sociodemograph	nic_data
	A data frame. The distribution of sociodemographic characteristics of the pop- ulation in the study area cells. Must contain the columns id and any others specified in population and socioeconomic_groups.
opportunity	A string. The name of the column in accessibility_data with the accessibil- ity levels to be considerend when calculating inequality levels.
population	A string. The name of the column in sociodemographic_data with the num- ber of people in each cell. Used to weigh accessibility levels when calculating inequality.
socioeconomic_	groups
	A string. The name of the column in sociodemographic_data whose values identify the socioeconomic groups that should be used to calculate the between- and within-groups inequality levels. If NULL (the default), between- and within- groups components are not calculated and only the total aggregate inequality is returned.
group_by	A character vector. When not character(0) (the default), indicates the accessibility_data columns that should be used to group the inequality estimates by. For example, if accessibility_data includes a scenario column that identifies distinct scenarios that each accessibility estimates refer to (e.g. before and after a transport policy intervention), passing "scenario" to this parameter results in inequality estimates grouped by scenario.

Value

If socioeconomic_groups is NULL, a data frame containing the total Theil T estimates for the study area. If not, a list containing three dataframes: one summarizing the total inequality and the between- and within-groups components, one listing the contribution of each group to the between-groups component and another listing the contribution of each group to the within-groups component.

See Also

Other inequality: concentration_index(), gini_index(), palma_ratio()

```
data_dir <- system.file("extdata", package = "accessibility")
travel_matrix <- readRDS(file.path(data_dir, "travel_matrix.rds"))
land_use_data <- readRDS(file.path(data_dir, "land_use_data.rds"))
access <- cumulative_cutoff(
   travel_matrix,
   land_use_data,</pre>
```

theil_t

```
cutoff = 30,
  opportunity = "jobs",
  travel_cost = "travel_time"
)
ti <- theil_t(</pre>
  access,
  sociodemographic_data = land_use_data,
  opportunity = "jobs",
  population = "population"
)
ti
# to calculate inequality between and within income deciles, we pass
# "income_decile" to socioeconomic_groups.
# some cells, however, are classified as in the decile NA because their
# income per capita is NaN, as they don't have any population. we filter
# these cells from our accessibility data, otherwise the output would include
# NA values (note that subsetting the data like this doesn't affect the
# assumption that groups are completely exhaustive, because cells with NA
# income decile don't have any population)
na_decile_ids <- land_use_data[is.na(land_use_data$income_decile), ]$id</pre>
access <- access[! access$id %in% na_decile_ids, ]</pre>
sociodem_data <- land_use_data[! land_use_data$id %in% na_decile_ids, ]</pre>
ti <- theil_t(</pre>
  access,
  sociodemographic_data = sociodem_data,
  opportunity = "jobs",
  population = "population",
  socioeconomic_groups = "income_decile"
)
ti
```

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