Package 'meshed'

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Title Bayesian Regression with Meshed Gaussian Processes

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Description

Fits Bayesian regression models based on latent Meshed Gaussian Processes (MGP) as described in Peruzzi, Banerjee, Finley (2020) <doi:10.1080/01621459.2020.1833889>, Peruzzi, Banerjee, Dunson, and Finley (2021) <arXiv:2101.03579>, Peruzzi and Dunson (2022) <arXiv:2201.10080>. Funded by ERC grant 856506 and NIH grant R01ES028804.

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meshed-package

Methods for fitting models based on Meshed Gaussian Processes (MGPs)

Description

meshed is a flexible package for Bayesian regression analysis on spatial or spatiotemporal datasets. The main function for fitting regression models is spmeshed, which outputs posterior samples obtained from Markov chain Monte Carlo which can be summarised using standard tools. The package also provides a function rmeshedgp for quickly simulating correlated spatial or spatiotemporal data at a very large number of locations.

Details

The functions rmeshedgp and spmeshed are provided for prior and posterior sampling (respectively) of Bayesian spatial or spatiotemporal multivariate regression models based on Meshed Gaussian Processes as introduced by Peruzzi, Banerjee, and Finley (2020). Posterior sampling via spmeshed proceeds by default via GriPS as detailed in Peruzzi, Banerjee, Dunson, and Finley (2021). When at least one outcome is not modeled with Gaussian errors, sampling proceeds taking advantage of Metropolis-adjusted Langevin dynamics as detailed in Peruzzi and Dunson (2022).

Author(s)

Michele Peruzzi

References

Peruzzi, M., Banerjee, S., and Finley, A.O. (2022) Highly Scalable Bayesian Geostatistical Modeling via Meshed Gaussian Processes on Partitioned Domains. *Journal of the American Statistical Association*, 117(538):969-982. doi:10.1080/01621459.2020.1833889

Peruzzi, M., Banerjee, S., Dunson, D.B., and Finley, A.O. (2021) Grid-Parametrize-Split (GriPS) for Improved Scalable Inference in Spatial Big Data Analysis. https://arxiv.org/abs/2101. 03579

Peruzzi, M., Dunson, D.B. (2022) Spatial meshing for general Bayesian multivariate models. https: //arxiv.org/abs/2201.10080

See Also

spmeshed, rmeshedgp

predict.spmeshed

Description

Sample from the posterior predictive distribution of the outcomes at new spatial or spatiotemporal locations after MCMC.

Usage

Arguments

object	Object output from spmeshed with option settings\$saving=TRUE.
newx	matrix of covariate values at the new coordinates.
newcoords	matrix of new coordinates.
n_threads	integer number of OpenMP threads. This is ineffective if meshed was not compiled with OpenMP support.
verbose	boolean for progress messagging.
	other arguments (unused).

Details

While this function can always be used to make predictions, in most cases it is more efficient to just include the prediction locations in the main data as NA values; spmeshed will sample from the posterior predictive distribution at those locations while doing MCMC. The predict method is only recommended when all 4 of the following are true:

(1) spmeshed was run with settings\$forced_grid=FALSE and

(2) the prediction locations are uniformly scattered on the domain (or rather, they are not clustered as a large empty area) and

(3) the number of prediction locations is a large portion of the number of observed data points and

(4) the prediction locations are not on a grid.

In all other cases the main spmeshed function is setup to be more efficient in automatically performing predictions during MCMC.

Value

coords_out	matrix with the prediction location coordinates (order updated after predictions).
preds_out	array of dimension (n_o, q, m) where n_o is the number of prediction locations, q
	is the output dimension, m is the number of MCMC samples.

Author(s)

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References

Peruzzi, M., Banerjee, S., and Finley, A.O. (2020) Highly Scalable Bayesian Geostatistical Modeling via Meshed Gaussian Processes on Partitioned Domains. *Journal of the American Statistical Association*, in press. doi:10.1080/01621459.2020.1833889

Peruzzi, M., Banerjee, S., Dunson, D.B., and Finley, A.O. (2021) Grid-Parametrize-Split (GriPS) for Improved Scalable Inference in Spatial Big Data Analysis. https://arxiv.org/abs/2101. 03579

```
# toy example with tiny dataset and short MCMC
# on a univariate outcome
library(magrittr)
library(dplyr)
library(meshed)
set.seed(2021)
SS <- 12
n <- SS^2 # total n. locations, including missing ones
coords <- data.frame(Var1=runif(n), Var2=runif(n)) %>%
  as.matrix()
# generate data
sigmasq <- 2.3
phi <- 6
tausq <- .1
B <- c(-1, .5, 1)
CC <- sigmasq * exp(-phi * as.matrix(dist(coords)))</pre>
LC <- t(chol(CC))</pre>
w <- LC %*% rnorm(n)</pre>
p <- length(B)
X <- rnorm(n * p) %>% matrix(ncol=p)
y_full <- X %*% B + w + tausq^.5 * rnorm(n)</pre>
set_missing <- rbinom(n, 1, 0.1)</pre>
simdata <- data.frame(coords,</pre>
                       y_full = y_full,
                       w_latent = w) \%>\%
  mutate(y_observed = ifelse(set_missing==1, NA, y_full))
# MCMC setup
mcmc_keep <- 500</pre>
```

rmeshedgp

```
mcmc_burn <- 100</pre>
mcmc_thin <- 2</pre>
y <- simdata$y_observed</pre>
ybar <- mean(y, na.rm=TRUE)</pre>
# training set
y_in <- (y-ybar)[!is.na(y)]</pre>
X_in <- X[!is.na(y),]</pre>
coords_in <- coords[!is.na(y),]</pre>
# suppose we dont want to have gridded knots
# i.e. we are fixing the MGP reference set at the observed locations
# (this may be inefficient in big data settings)
meshout <- spmeshed(y_in, X_in, coords_in,</pre>
                     axis_partition=c(4,4),
                     n_samples = mcmc_keep,
                     n_burn = mcmc_burn,
                     n_thin = mcmc_thin,
                      settings = list(forced_grid=FALSE, cache=FALSE),
                      prior=list(phi=c(1,15)),
                      verbose = 0,
                      n_{threads} = 1)
# test set
coords_out <- coords[is.na(y),]</pre>
X_out <- X[is.na(y),]</pre>
df_predict <- predict(meshout, newx=X_out, newcoords=coords_out)</pre>
y_posterior_predictive_mean <- df_predict$preds_out[,1,] %>%
  apply(1, mean) %>% add(ybar)
df_predicted <- df_predict$coords_out %>% cbind(y_posterior_predictive_mean)
```

rmeshedgp

```
Prior sampling from a Meshed Gaussian Process
```

Description

Generates samples from a (univariate) MGP assuming a cubic directed acyclic graph and axisparallel domain partitioning.

Usage

```
rmeshedgp(coords, theta,
    axis_partition = NULL, block_size = 100,
    n_threads=1, cache=TRUE, verbose=FALSE, debug=FALSE)
```

Arguments

coords	matrix of spatial or spatiotemporal coordinates with $d = 2$ or $d = 3$ columns for spatial or spatiotemporal data, respectively.
theta	vector with covariance parameters. If $d = 2$ and theta is a 2-dimensional vector then $\theta = (\phi, \sigma^2)$ where ϕ is the spatial decay and σ^2 is the spatial variance in the exponential covariance model. If $d = 2$ and theta is a 3-dimensional vector then $\theta = (\phi, \nu, \sigma^2)$ and a Matern model with smoothness ν is used instead. If $d = 3$, theta must be a 4-dimensional vector and $\theta = (a, \phi, b, \sigma^2)$ using Gneiting's non-separable spatiotemporal covariance detailed below.
axis_partition	integer vector of length d with the number of intervals along which each axis should be partitioned. The domain will be partitioned into prod(axis_partition) blocks. This argument can be left blank when using block_size.
block_size	integer specifying the (approximate) size of the blocks, i.e. how many spatial or spatiotemporal locations should be included in each block. Note: larger values correspond to an MGP that is closer to a full GP, but require more expensive computations.
n_threads	integer number of OpenMP threads. This is ineffective if meshed was not com- piled with OpenMP support.
cache	bool: whether to use cache. Some computational speedup is associated to cache=TRUE if coords are a grid.
verbose	bool: print some messages.
debug	bool: print more messages.

Details

Gaussian processes (GPs) lack in scalability to big datasets due to the assumed unrestricted dependence across the spatial or spatiotemporal domain. *Meshed* GPs instead use a directed acyclic graph (DAG) with patterns, called *mesh*, to simplify the dependence structure across the domain. Each DAG node corresponds to a partition of the domain. MGPs can be interpreted as approximating the GP they originate from, or as standalone processes that can be sampled from. This function samples random MGPs and can thus be used to generate big spatial or spatiotemporal data. The only requirement to sample from a MGP compared to a standard GP is the specification of the domain partitioning strategy. Here, either axis_partition or block_size can be used; the default block_size=100 can be used to quickly sample smooth surfaces at millions of locations.

Just like in a standard GP, one needs a covariance function or kernel which can be set as follows. For spatial data (d = 2), the length of theta determines which model is used (see above). Letting h = ||s - s'|| where s and s' are locations in the spatial domain, the exponential covariance is defined as:

$$C(h) = \sigma^2 \exp\{-\phi h\},\$$

whereas the Matern model is

$$C(h) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \phi^{\nu} h^{\nu} K_{\nu}(\phi h),$$

where K_{ν} is the modified Bessel function of the second kind of order ν . For spatiotemporal data (d = 3) the covariance function between locations (s, t) and (s', t') with distance h = ||s - s'|| and

rmeshedgp

time lag u = ||t - t'|| is defined as

 $C(h, u) = \sigma^2 / (au + 1) \exp\{-\phi h(au + 1)^{-b/2}\},\$

which is a special case of non-separable spacetime covariance as introduced by Gneiting (2002).

Value

data.frame with the (reordered) supplied coordinates in the first d columns, and the MGP sample in the last column, labeled w.

Author(s)

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References

Gneiting, T (2002) Nonseparable, Stationary Covariance Functions for Space-Time Data. *Journal of the American Statistical Association*. doi:10.1198/016214502760047113

Peruzzi, M., Banerjee, S., and Finley, A.O. (2020) Highly Scalable Bayesian Geostatistical Modeling via Meshed Gaussian Processes on Partitioned Domains. *Journal of the American Statistical Association*, in press. doi:10.1080/01621459.2020.1833889

```
library(ggplot2)
library(magrittr)
library(meshed)
# spatial domain (we choose a grid to make a nice image later)
# this generates a dataset of size 6400
xx <- seq(0, 1, length.out=80)</pre>
coords <- expand.grid(xx, xx) %>%
  as.matrix()
raster_plot <- function(df){</pre>
  ggplot(df, aes(Var1, Var2, fill=w)) +
    geom_raster() +
    scale_fill_viridis_c() +
    theme_minimal() }
# spatial data, exponential covariance
# phi=14, sigma^2=2
simdata <- rmeshedgp(coords, c(14, 2))</pre>
raster_plot(simdata)
# spatial data, matern covariance
# phi=14, nu=1, sigma^2=2
simdata <- rmeshedgp(coords, c(14, 1, 2))</pre>
raster_plot(simdata)
```

```
# spacetime data, gneiting's covariance
# 64000 locations
stcoords <- expand.grid(xx, xx, seq(0, 1, length.out=10))
# it should take less than a couple of seconds
simdata <- rmeshedgp(stcoords, c(1, 14, .5, 2))
# plot data at 7th time period
raster_plot(simdata %>% dplyr::filter(Var3==unique(Var3)[7]))
```

spmeshed

Posterior sampling for models based on MGPs

Description

Fits Bayesian multivariate spatial or spatiotemporal regression models with latent MGPs via Markov chain Monte Carlo.

Usage

```
spmeshed(y, x, coords, k=NULL,
       family = "gaussian",
       axis_partition = NULL,
       block_size = 30,
       grid_size = NULL,
       grid_custom = NULL,
       n_{samples} = 1000,
       n_{burnin} = 100,
       n_{thin} = 1,
       n_{threads} = 4,
       verbose = 0,
       predict_everywhere = FALSE,
       settings = list(adapting=TRUE, forced_grid=NULL,
                        cache=NULL, ps=TRUE, saving=TRUE, low_mem=FALSE, hmc=4),
       prior = list(beta=NULL, tausq=NULL, sigmasq = NULL,
                          phi=NULL, a=NULL, nu = NULL,
                          toplim = NULL, btmlim = NULL, set_unif_bounds=NULL),
       starting = list(beta=NULL, tausq=NULL, theta=NULL, lambda=NULL, v=NULL,
                       a=NULL, nu = NULL,
                       mcmcsd=.05,
                       mcmc_startfrom=0),
       debug = list(sample_beta=TRUE, sample_tausq=TRUE,
                    sample_theta=TRUE, sample_w=TRUE, sample_lambda=TRUE,
                    verbose=FALSE, debug=FALSE),
       indpart=FALSE
)
```

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Arguments

У	matrix of multivariate outcomes with n rows and q columns. Each row of y corresponds to a row of coords. NA values are accepted in any combination and will be predicted via MCMC.
х	matrix of covariates with n rows and p columns.
coords	matrix of coordinates with n rows and $d = 2$ or $d = 3$ columns for spatial or spacetime regression, respectively.
k	integer $k \leq q$, number of latent processes to use for the linear model of core- gionalization. If unspecified, this is set to $q=ncol(y)$.
family	a vector with length 1 or q whose elements corresponds to the data types of columns of y. Available choices are gaussian, poisson, binomial, beta for outcomes that are continuous, count, binary, or $(0, 1)$ proportions.
axis_partition	integer vector of size d: number of intervals each coordinate axis is split into
block_size	integer approximate size of the blocks after domain partitioning. Only used if axis_partition is not specified.
grid_size	integer vector of size d: number of 'knots' of the reference grid along each axis. This grid is then partitioned using either axis_partition or block_size. If unspecified, this is set so that the eventual grid size is close to n. This parameter is ignored if settings\$forced_grid=FALSE in which case the data are assumed to be on a grid.
grid_custom	list with elements grid and axis_interval_partition. grid is a data.frame with the user supplied grid of knots. It is possible to include covariate values for the grid locations as additional columns, as long as their number matches ncol(x) - this is useful to make raster images of predictions. axis_interval_partition is the user supplied set of cuts for each coordinate axis (Note: these are the actual cutpoints along the axes, not the number of cuts). If left empty, axis_partition will be used to partition the custom grid. No checks are made on the validity of this grid. This parameter is ignored if settings\$forced_grid=FALSE in which case the data are assumed to be on a grid.
n_samples	integer number of MCMC samples at which all the unknowns are stored (in- cluding the latent effects).
n_burnin	integer number of MCMC samples to discard at the beginning of the chain.
n_thin	integer thinning parameter for the MCMC chain. Only the chain of latent effects (w) is thinned to save memory in big data problems. Chains for other unknowns are not thinned and thus will be of length n_thin * n_samples.
n_threads	integer number of OpenMP threads. This is ineffective if meshed was not com- piled with OpenMP support.
verbose	integer. If verbose<=20, then this is the number of times a message is displayed during MCMC. If verbose>20, then this is the number of MCMC iterations to wait until the next message update. If verbose=Inf, then a message will be printed at each MCMC iteration.
predict_everyw	
	bool used if settings\$forced_grid=T. Should predictions be made at the reference grid locations? If not, predictions will be made only at the supplied NA values of Y.

settings	list: settings\$adapting turns the adaptation of MCMC on/off, settings\$forced_grid determines whether or not to use the data grid or a forced grid; if unspecified, the
	function will try to see what the data look like. Note: if forced_grid=FALSE
	and n is very large and $coords$ are irregularly spaced, then expect slowdowns in
	preprocessing and consider using forced_grid=TRUE instead. settings\$saving
	will save model data if set to TRUE. settings\$low_mem will only save beta_mcmc,
	lambda_mcmc, v_mcmc, tausq_mcmc (and not w_mcmc and lp_mcmc, which can
	be recovered from the others), thereby using less memory. All fitted predic-
	tions remain available in yhat_mcmc for convenience. settings\$ps (default
	TRUE) determines whether to use the PS parametrization (Peruzzi et al 2021).
	settings\$hmc, used if any outcome is not Gaussian, (1: MALA, 2: NUTS, 3:
	RM-MALA, 4: Simplified manifold preconditioning (default))
prior	list: setup for priors of unknown parameters. prior\$phi needs to be specified as the support of the Uniform prior for ϕ . There is currently limited functionality
	here and some inputs are currently ignored. Defaults are: a vague Gaussian for β , $\tau_i^2 \sim IG(2,1)$, $\theta_j \sim IG(2,2)$, all subject to change.
starting	list: setup for starting values of unknown parameters. starting\$mcmcsd is
	the initial standard deviation of proposals. starting\$mcmc_startfrom is input
	to the adaptive MCMC and can be used to manually restart MCMC. There is currently limited functionality here and some parameters may be ignored.
debug	list: setup for debugging things. Some parts of MCMC can be turned off here.
indpart	bool defaults to FALSE. If TRUE, this computes an independent partition model.

Details

This function targets the following model:

 $y(s) = x(s)^{\top}\beta + \Lambda v(s) + \epsilon(s),$

where y(s) is a q-dimensional vector of outcomes at spatial location s, x(s) is a p-dimensional vector of covariates with static coefficients β , Λ is a matrix of factor loadings of size (q, k), v(s) is a k-dimensional vector which collects the realization of independent Gaussian processes $v_j \sim spmeshed(0, C_j)$ for $j = 1, \ldots, k$ and where $C_j(s, s')$ is a correlation function. s is a coordinate in space (d = 2) or space plus time (d = 3). The Meshed GP implemented here associates an axis-parallel tessellation of the domain to a cubic directed acyclic graph (mesh).

Value

coordsdata	data.frame including the original n coordinates plus the n_g knot coordinates if the model was run on a forced grid. The additional column forced_grid has value 1 if the corresponding coordinate is a knot in the forced grid. See examples.
savedata	Available if settings\$saving==TRUE. Needed for making predictions using predict() after MCMC. Note: NA values of the output are automatically and more efficiently predicted when running spmeshed.
yhat_mcmc	list of length n_samples whose elements are matrices with $n + n_g$ rows and q columns. Each matrix in the list is a posterior predictive sample of the latent spatial process. $n_g = 0$ if the data grid is being used. Given the possibly large n , only the thinned chain is output for y .

v_mcmc	list of length n_samples whose elements are matrices with $n + n_g$ rows and k columns. Each matrix in the list is a posterior sample of the k latent spatial process. $n_g = 0$ if the data grid is being used. Given the possibly large n , only the thinned chain is output for v .
w_mcmc	list of length n_samples whose elements are matrices with $n + n_g$ rows and q columns. Each matrix in the list is a posterior sample of $w = \Lambda v$. $n_g = 0$ if the data grid is being used. Given the possibly large n , only the thinned chain is output for w .
lp_mcmc	list of length n_samples whose elements are matrices with $n + n_g$ rows and q columns. Each matrix in the list is a posterior sample of the linear predictor $X\beta + \Lambda v$. $n_g = 0$ if the data grid is being used. Given the possibly large n , only the thinned chain is output for w .
beta_mcmc	array of size (p, q, n_thin*n_samples) with the posterior sample for the static regression coefficients β . The <i>j</i> th column of each matrix (<i>p</i> rows and <i>q</i> columns) corresponds to the <i>p</i> linear effects on the <i>j</i> th outcome. The full chain minus burn-in is returned NOT thinned since p and q are relatively small.
tausq_mcmc	matrix of size (q, n_thin*n_samples). Each row corresponds to the full MCMC chain for the nugget τ_j^2 of the <i>j</i> th outcome in the coregionalization/factor model. The full chain minus burn-in is returned NOT thinned since q is relatively small.
theta_mcmc	array of size (h, k, n_thin*n_samples) with the posterior sample for the correlation function parameters θ . h is 2 for spatial data (corresponding to the spatial decay of the exponential covariance $(\phi_i, i = 1, \ldots, k)$, and the variance $\sigma_i^2, i = 1, \ldots, k$), 4 for spacetime data (corresponding to temporal decay, spatial decay, and separability – these are referred to as a_i, ϕ_i , and $\beta_i, i = 1, \ldots, k$, in Gneiting (2002), see doi:10.1198/016214502760047113, plus the variance $\sigma^2, i = 1, \ldots, k$). The full chain minus burn-in is returned NOT thinned since h and k are relatively small. If settings\$ps=TRUE, the MCMC output for σ_i^2 (last row of theta_mcmc) should be discarded and Λ used instead.
lambda_mcmc	array of size (q, k, n_thin*n_samples). Each matrix (of size (q, k)) is a pos- terior sample for Λ in the coregionalization/factor model. In univariate models, this is usually called σ . The full chain minus burn-in is returned NOT thinned since q and k are relatively small.
paramsd	Cholesky factorization of the proposal covariance for adaptive MCMC, after adaptation.
mcmc	Total number of MCMC iterations performed.
<pre>mcmc_time</pre>	Time in seconds taken for MCMC (not including preprocessing).

Author(s)

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References

Peruzzi, M., Banerjee, S., and Finley, A.O. (2022) Highly Scalable Bayesian Geostatistical Modeling via Meshed Gaussian Processes on Partitioned Domains. *Journal of the American Statistical Association*, 117(538):969-982. doi:10.1080/01621459.2020.1833889

Peruzzi, M., Banerjee, S., Dunson, D.B., and Finley, A.O. (2021) Grid-Parametrize-Split (GriPS) for Improved Scalable Inference in Spatial Big Data Analysis. https://arxiv.org/abs/2101. 03579

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```
# toy example with tiny dataset and short MCMC
# on a univariate outcome
library(magrittr)
library(dplyr)
library(ggplot2)
library(meshed)
set.seed(2021)
SS <- 12
n <- SS^2 # total n. locations, including missing ones</pre>
coords <- expand.grid(xx <- seq(0,1,length.out=SS), xx) %>%
  as.matrix()
# generate data
sigmasq <- 2.3
phi <- 6
tausq <- .1
B <- c(-1,.5,1)
CC <- sigmasq * exp(-phi * as.matrix(dist(coords)))</pre>
LC <- t(chol(CC))
w <- LC %*% rnorm(n)</pre>
p <- length(B)
X <- rnorm(n * p) %>% matrix(ncol=p)
y_full <- X %*% B + w + tausq^.5 * rnorm(n)</pre>
set_missing <- rbinom(n, 1, 0.1)</pre>
simdata <- data.frame(coords,</pre>
                       y_full = y_full,
                        w_latent = w) \%>\%
  mutate(y_observed = ifelse(set_missing==1, NA, y_full))
# MCMC setup
mcmc_keep <- 500</pre>
mcmc_burn <- 100</pre>
mcmc_thin <- 2
y <- simdata$y_observed</pre>
ybar <- mean(y, na.rm=TRUE)</pre>
```

```
meshout <- spmeshed(y-ybar, X, coords,</pre>
                    axis_partition=c(4,4),
                    n_samples = mcmc_keep,
                    n_burn = mcmc_burn,
                    n_thin = mcmc_thin,
                    prior=list(phi=c(1,15)),
                    verbose = 0,
                    n_{threads} = 1)
# posterior means
best_post_mean <- meshout$beta_mcmc %>% apply(1:2, mean)
# process means
wmesh <- data.frame(w_mgp = meshout$w_mcmc %>% summary_list_mean())
# predictions
ymesh <- data.frame(y_mgp = meshout$yhat_mcmc %>% summary_list_mean())
outdf <-
 meshout$coordsdata %>%
 cbind(ymesh, wmesh)
# plot predictions
pred_plot <- outdf %>%
 ggplot(aes(Var1, Var2, color=y_mgp)) +
 geom_point() +
 scale_color_viridis_c()
# plot latent process
latent_plot <- outdf %>%
 ggplot(aes(Var1, Var2, color=w_mgp)) +
 geom_point() +
 scale_color_viridis_c()
# estimation of regression coefficients
plot(density(meshout$beta_mcmc[1,1,]))
abline(v=B[1], col="red")
```

summary_list_mean Arithmetic mean of matrices in a list

Description

For a list of matrices $\{X^{(1)}, \ldots, X^{(L)}\}$, all of the same dimension, this function computes the matrix \bar{X} with i, j entry $\bar{X}_{i,j} = \frac{1}{L} \sum_{l=1}^{L} X_{i,j}^{(l)}$. This function does not run any check on the dimensions and uses OpenMP if available.

Usage

```
summary_list_mean(x, n_threads=1)
```

Arguments

х	A list of matrices of the same dimension
n_threads	integer number of OpenMP threads. This is ineffective if meshed was not com- piled with OpenMP support.

Value

The matrix of mean values.

Author(s)

Michele Peruzzi <michele.peruzzi@duke.edu>

Examples

```
# make some data into a list
set.seed(2021)
L <- 200
x <- lapply(1:L, function(i) matrix(runif(300), ncol=3))
mean_done <- summary_list_mean(x)</pre>
```

summary_list_q Quantiles of elements of matrices in a list

Description

For a list of matrices $\{X^{(1)}, \ldots, X^{(L)}\}$, all of the same dimension, this function computes the matrix \hat{X} with i, j entry $\hat{X}_{i,j} = \text{quantile}(\{X_{i,j}^{(l)}\}_{l=1}^{L}, q)$. This function does not run any check on the dimensions and uses OpenMP if available. This is only a convenience function that is supposed to speed up quantile computation for very large problems. The results may be slightly different from R's quantile which should be used for small problems.

Usage

summary_list_q(x, q, n_threads=1)

Arguments

х	A list of matrices of the same dimension.
q	A number between 0 and 1.
n_threads	integer number of OpenMP threads. This is ineffective if meshed was not com- piled with OpenMP support.

Value

The matrix of quantiles.

summary_list_q

Author(s)

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```
# make some data into a list
set.seed(2021)
L <- 200
x <- lapply(1:L, function(i) matrix(runif(300), ncol=3))
quant_done1 <- summary_list_q(x, .9)</pre>
```

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