

Package ‘capn’

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Description

Implements approximation methods for natural capital asset prices suggested by Fenichel and Abbott (2014) <[doi:10.1086/676034](https://doi.org/10.1086/676034)> in Journal of the Associations of Environmental and Resource Economists (JAERE), Fenichel et al. (2016) <[doi:10.1073/pnas.1513779113](https://doi.org/10.1073/pnas.1513779113)> in Proceedings of the National Academy of Sciences (PNAS), and Yun et al. (2017) in PNAS (accepted), and their extensions: creating Chebyshev polynomial nodes and grids, calculating basis of Chebyshev polynomials, approximation and their simulations for: V-approximation (single and multiple stocks, PNAS), P-approximation (single stock, PNAS), and Pdot-approximation (single stock, JAERE). Development of this package was generously supported by the Knobloch Family Foundation.

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aproxdef	<i>Defining Approximation Space</i>
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Description

The function defines an approximation space for all three approximation apporoaches (V, P, and Pdot).

Usage

aproxdef(deg, lb, ub, delta)

Arguments

deg	An array of degrees of approximation function: degrees of Chebyshev polynomials
lb	An array of lower bounds
ub	An array of upper bounds
delta	discount rate

Details

For the i -th dimension of $i = 1, 2, \dots, d$, suppose a polynomial approximant s_i over a bounded interval $[a_i, b_i]$ is defined by Chebysev nodes. Then, a d -dimensional Chebyshev grids can be defined as:

$$\mathbf{S} = \{(s_1, s_2, \dots, s_d) | a_i \leq s_i \leq b_i, i = 1, 2, \dots, d\}.$$

Suppose we impletement n_i numbers of polynomials (i.e., $(n_i - 1)$ -th order) for the i -th dimension. The approximation space is defined as:

$$\begin{aligned} \text{deg} &= \mathbf{c}(n_1, n_2, \dots, n_d), \\ \text{lb} &= \mathbf{c}(a_1, a_2, \dots, a_d), \text{ and} \\ \text{ub} &= \mathbf{c}(b_1, b_2, \dots, b_d). \end{aligned}$$

delta is the given constant discount rate.

Value

A list containing the approximation space

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[vapprox](#), [vsim](#), [papprox](#), [psim](#), [pdotapprox](#), [pdotsim](#)

Examples

```
## Reef-fish example: see Fenichel and Abbott (2014)
delta <- 0.02
upper <- 359016000      # upper bound on approximation space
lower <- 5*10^6          # lower bound on approximation space
myspace <- aproxdef(50,lower,upper,delta)
## Two dimensional example
ub <- c(1.5,1.5)
lb <- c(0.1,0.1)
deg <- c(20,20)
delta <- 0.03
myspace <- aproxdef(deg,lb,ub,delta)
```

catch	<i>catch function of GOM dataset</i>
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Description

The function calculates the catchment in the reef-fishy example of GOM dataset (Fenichel and Abbott, 2014).

Usage

catch(s,Z)

Arguments

s	stock
Z	parameter vector

Details

This catch function is adopted in GOM dataset.

Value

Quantity of catchment

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

chebbasisgen	<i>Generating Unidimensional Chebyshev polynomial (monomial) basis</i>
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Description

The function calculates the monomial basis of Chebyshev polynomials for the given unidimensional nodes, s_i , over a bounded interval [a,b].

Usage

chebbasisgen(stock, npol, a, b, dorder = NULL)

Arguments

stock	An array of Chebyshev polynomial nodes s_i (an array of stocks in capn-packages)
npol	Number of polynomials (n polynomials = (n-1)-th degree)
a	The lower bound of interval [a,b]
b	The upper bound of interval [a,b]
dorder	Degree of partial derivative of the basis; Default is NULL; if dorder = 1, returns the first order partial derivative

Details

Suppose there are m numbers of Chebyshev nodes over a bounded interval $[a,b]$:

$$s_i \in [a, b], \text{ for } i = 1, 2, \dots, m.$$

These nodes can be nomralized to the standard Chebyshev nodes over the domain $[-1,1]$:

$$z_i = \frac{2(s_i - a)}{(b - a)} - 1.$$

With normalized Chebyshev nodes, the recurrence relations of Chebyshev polynomials of order n is defined as:

$$\begin{aligned} T_0(z_i) &= 1, \\ T_1(z_i) &= z_i, \text{ and} \\ T_n(z_i) &= 2z_i T_{n-1}(z_i) - T_{n-2}(z_i). \end{aligned}$$

The interpolation matrix (Vandermonde matrix) of (n-1)-th Chebyshev polynomials with m nodes, Φ_{mn} is:

$$\Phi_{mn} = \begin{bmatrix} 1 & T_1(z_1) & \cdots & T_{n-1}(z_1) \\ 1 & T_1(z_2) & \cdots & T_{n-1}(z_2) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & T_1(z_m) & \cdots & T_{n-1}(z_m) \end{bmatrix}.$$

The partial derivative of the monomial basis matrix can be found by the relation:

$$(1 - z_i^2)T'_n(z_i) = n[T_{n-1}(z_i) - z_i T_n(z_i)].$$

The technical details of the monomial basis of Chebyshev polynomial can be referred from Amparo et al. (2007) and Miranda and Fackler (2012).

Value

A matrix (number of nodes (m) x npol (n)) of (monomial) Chebyshev polynomial basis

References

Amparo, Gil, Javier Segura, and Nico Temme. (2007) *Numerical Methods for Special Functions*. Cambridge: Cambridge University Press.
 Miranda, Mario J. and Paul L. Fackler. (2002) *Applied Computational Economics and Finance*. Cambridge: The MIT Press.

See Also

[chebnodegen](#)

Examples

```
## Reef-fish example: see Fenichel and Abbott (2014)
data("GOM")
nodes <- chebnodegen(param$nodes,param$lowerK,param$upperK)
## An example of Chebyshev polynomial basis
chebbasisgen(nodes,20,0.1,1.5)
## The partial derivative of Chebyshev polynomial basis with the same function
chebbasisgen(nodes,20,0.1,1.5,1)
```

chebgrids

Generating Chebyshev grids

Description

This function generates a grid of multi-dimensional Chebyshev nodes.

Usage

```
chebgrids(nnodes, lb, ub, rtype = NULL)
```

Arguments

nnodes	An array of numbers of nodes
lb	An array of lower bounds
ub	An array of upper bounds
rtype	A type of results; default is NULL that returns a list class; if rtype = list, returns a list class; if rtype = grid, returns a matrix class.

Details

For the i -th dimension of $i = 1, 2, \dots, d$, suppose a polynomial approximant s_i over a bounded interval $[a_i, b_i]$ is defined by Chebyshev nodes. Then, a d -dimensional Chebyshev grids can be defined as:

$$\mathbf{S} = \{(s_1, s_2, \dots, s_d) | a_i \leq s_i \leq b_i, i = 1, 2, \dots, d\}.$$

This is all combinations of s_i . Two types of results are provided. 'rtype = list' provides a list of d dimensions whereas 'rtype = grids' creates a $\left(\prod_{i=1}^d n_i\right) \times d$ matrix.

Value

A list with d elements of Chebyshev nodes or a $\left(\prod_{i=1}^d n_i\right) \times d$ matrix of Chebyshev grids

See Also

[chebnodegen](#)

Examples

```
## Chebyshev grids with two-dimension
chebgrids(c(5,3), c(1,1), c(2,3))
# Returns the same results
chebgrids(c(5,3), c(1,1), c(2,3), rtype='list')
## Returns a matrix grids with the same domain
chebgrids(c(5,3), c(1,1), c(2,3), rtype='grid')
## Chebyshev grids with one-dimension
chebgrids(5,1,2)
chebnodegen(5,1,2)
## Chebyshev grids with three stock
chebgrids(c(3,4,5),c(1,1,1),c(2,3,4),rtype='grid')
```

chebnodegen	<i>Unidimensional Chebyshev nodes</i>
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Description

The function generates uni-dimensional chebyshev nodes.

Usage

```
chebnodegen(n, a, b)
```

Arguments

n	A number of nodes
a	The lower bound of interval [a,b]
b	The upper bound of interval [a,b]

Details

A polynomial approximant s_i over a bounded interval $[a,b]$ is constructed by:

$$s_i = \frac{b+a}{2} + \frac{b-a}{2} \cos\left(\frac{n-i+0.5}{n}\pi\right) \text{ for } i = 1, 2, \dots, n.$$

More detail explanation can be referred from Miranda and Fackler (2002, p.119).

Value

An array n Chebyshev nodes

References

Miranda, Mario J. and Paul L. Fackler. (2002) *Applied Computational Economics and Finance*. Cambridge: The MIT Press.

Examples

```
## 10 Chebyshev nodes in [-1,1]
chebnodegen(10,-1,1)
## 5 Chebyshev nodes in [1,5]
chebnodegen(5,1,5)
```

dsdotds

first derivative function of sdot in GOM dataset

Description

dsdotds evaluated $\frac{dsdot}{ds}$ in the reef-fishy example of GOM dataset (Fenichel and Abbott, 2014).

Usage

```
dsdotds(s,Z)
```

Arguments

s	stock
Z	parameter vector

Details

This function is adopted in GOM dataset.

Value

The first derivative of sdot with respect to s

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

dsdotdss	<i>second derivative function of sdot in GOM dataset</i>
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Description

dsdotdss evaluated $\frac{d}{ds}(\frac{dsdot}{ds})$ in the reef-fishy example of GOM dataset (Fenichel and Abbott, 2014).

Usage

dsdotdss(s,Z)

Arguments

s	stock
Z	parameter vector

Details

This function is adopted in GOM dataset.

Value

The second derivative of sdot with respect to s

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

dwds	<i>first derivative function of profit in GOM dataset</i>
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Description

dwds evaluated $\frac{dw}{ds}$ in the reef-fishy example of GOM dataset (Fenichel and Abbott, 2014).

Usage

dwds(s,Z)

Arguments

- s stock
- Z parameter vector

Details

This function is adopted in GOM dataset.

Value

The first derivative of w with respect to s

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

dwdss	<i>second derivative function of profit in GOM dataset</i>
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Description

dwdss evaluated $\frac{d}{ds}(\frac{dw}{ds})$ in the reef-fishy example of GOM dataset (Fenichel and Abbott, 2014).

Usage

dwdss(s,Z)

Arguments

s	stock
Z	parameter vector

Details

This function is adopted in GOM dataset.

Value

The second derivative of w with respect to s

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

effort	<i>effort function of GOM dataset</i>
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Description

The function calculates the catchment effort in the reef-fishy example of GOM dataset (Fenichel and Abbott, 2014).

Usage

effort(s,Z)

Arguments

s	stock
Z	parameter vector

Details

This effort function is adopted in GOM dataset.

Value

catchment effort values

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "[Natural Capital: From Metaphor to Measurement.](#)" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

GOM

Reef Fish example: one dimensional stock

Description

The GOM provides data to replicate the Gulf of Mexico Reef Fish example in [Fenichel and Abbott \(2014\)](#). This dataset is consisted of parameters and functions. From Fenichel and Abboott(2014),

catch effort: $x(s) = ys^\gamma$,

harvest: $h(s, x) = q((ys^\gamma)^\alpha)s = q(y^\alpha)(s^{\gamma\alpha})$,

profit: $w(s, x) = price \cdot h(s, x) - cost \cdot x(s)$, and

sdot: $\dot{s} = rs \left(1 - \frac{s}{k}\right) - q(y^\alpha)(s^{\gamma\alpha+1})$.

The parameters in detal are in below.

Usage

```
## Load dataset
data("GOM")
## Demonstration of example
# demo(GOM, package="capn")
## R-script location
# system.file("demo", "GOM.R", package = "capn")
```

Format

param: a data.frame of parameters

- r intrinsic growth rate (=0.3847)
- k carrying capacity (=359016000)
- q catchability coefficient (=0.00031729344157311126)
- price price (=2.70)
- cost cost (=153.0)
- alpha technology parameter (=0.5436459179063678)

- gamma pre-ITQ management parameter (=0.7882)
- y system equivalence parameter (=0.15745573410462155)
- delta discount rate (=0.02)
- order Chebyshev polynomial order (=50)
- upperK upper bound of Chebyshev polynomial nodes (=k)
- lowerK lower bound of Chebyshev polynomial nodes (=5*10^6)
- nodes the number of Chebyshev polynomial nodes (=50)

functions: functions for generate simulation data for each nodes

- effort effort function
- catch catch function
- profit profit function (w in Fenichel and Abbott (2014))
- sdot evaluated $\frac{dst}{dt}$
- dsdotds evaluated $\frac{dsdot}{ds}$
- dsdotdss evaluated $\frac{d}{ds} \left(\frac{dsdot}{ds} \right)$
- dwds evaluated $\frac{dw}{ds}$
- dwdss evaluated $\frac{d}{ds} \left(\frac{dw}{ds} \right)$

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

LV

Prey-Predator (Lotka-Volterra) example: two stocks

Description

The LV provides the data and functions to simulate prey-predator (Lotka-Volterra) model. The original code was written by Joshua Abbott in MATLAB and Seong Do Yun adapted it to a package example. The prey-predator model is:

Prey (X): $\dot{X} = rX \left(1 - \frac{X}{K}\right) - aXY - \theta X$, and

Predator (Y): $\dot{Y} = bXY - mY - \gamma Y$.

The parameters are given as:

$r = 0.025$: intrinsic growth rate for prey,

$K = 1$: carrying capacity for prey,

$a = 0.08$: predator-related mortality parameter for prey,

$b = 0.05$: predator/prey uptake parameter for predator,

$m = 0.01$: natural mortality for predator,

$\gamma = 0.005$: slope for linear predator harvest control rule, and
 $\theta = 0.005$: slope for linear prey harvest control rule

The predator with no economic value (unharvested) is designed for the economic program as:

$$W = \text{harv.prey}(p.\text{prey} - c.\text{prey}/X)\theta X + \text{harv.pred} * (p.\text{pred} - c.\text{pred}/Y)\gamma Y.$$

The paramters are:

$p.\text{pred} = 0$: price per unit harvest of predator,

$p.\text{prey} = 25$: price per unit harvest of prey,

$c.\text{prey} = 0.1p.\text{prey}$: cost /per unit of prey effort in Schaefer model (really c/q with $q=1$), and

$c.\text{pred} = c.\text{prey}$: cost per unit of predator effort in Schaefer model (really c/q with $q=1$).

Usage

```
## Load dataset
data("lvdata")
## Demonstration of example
# demo(LV, package="capn")
## R-script location
# system.file("demo", "LV.R", package = "capn")
```

Format

lvaproxdata: a data.frame for approximation (evaluated on (20 x 20) Chebyshev nodes)

- xs prey stock
- ys predator stock
- xdot evaluated $\frac{dx}{dt}$
- ydot evaluated $\frac{dy}{dt}$
- wval profit (W in Fenichel and Abtott (2014))

lvsimdata.time: a data for time simulation (101 ODE solution)

- tseq time sequence from 0 to 100
- xs prey stock
- ys predator stock

Description

The `lvaproxdata` provides the data in LV dataset to simulate prey-predator (Lotka-Volterra) model. The original code was written by Joshua Abbott in MATLAB and Seong Do Yun adapted it to a package example. The prey-predator model is:

Prey (X): $\dot{X} = rX \left(1 - \frac{X}{K}\right) - aXY - \theta X$, and

Predator (Y): $\dot{Y} = bXY - mY - \gamma Y$.

The parameters are given as:

$r = 0.025$: intrinsic growth rate for prey,

$K = 1$: carrying capacity for prey,

$a = 0.08$: predator-related mortality parameter for prey,

$b = 0.05$: predator/prey uptake parameter for predator,

$m = 0.01$: natural mortality for predator,

$\gamma = 0.005$: slope for linear predator harvest control rule, and

$\theta = 0.005$: slope for linear prey harvest control rule

The predator with no economic value (unharvested) is designed for the economic program as:

$$W = \text{harv.prey}(p.\text{prey} - c.\text{prey}/X)\theta X + \text{harv.pred} * (p.\text{pred} - c.\text{pred}/Y)\gamma Y.$$

The paramters are:

$p.\text{pred} = 0$: price per unit harvest of predator,

$p.\text{prey} = 25$: price per unit harvest of prey,

$c.\text{prey} = 0.1p.\text{prey}$: cost /per unit of prey effort in Schaefer model (really c/q with $q=1$), and

$c.\text{pred} = c_p.\text{prey}$: cost per unit of predator effort in Schaefer model (really c/q with $q=1$).

Usage

```
## Load dataset
data("lvdata")
```

Format

lvaproxdata: a data.frame for approximation (evaluated on (20 x 20) Chebyshev nodes)

- `xs` prey stock
- `ys` predator stock
- `xdot` evaluated $\frac{dx}{dt}$
- `ydot` evaluated $\frac{dy}{dt}$
- `wval` profit (W in Fenichel and Abtott (2014))

See Also

[LV](#), [vsim](#)

lvsimdata.time

*Prey-Predator (Lotka-Volterra) example in LV dataset***Description**

The `lvsimdata.time` provides the time simulation data in LV dataset to simulate prey-predator (Lotka-Volterra) model. The original code was written by Joshua Abbott in MATLAB and Seong Do Yun adapted it to a package example. The prey-predator model is:

Prey (X): $\dot{X} = rX \left(1 - \frac{X}{K}\right) - aXY - \theta X$, and

Predator (Y): $\dot{Y} = bXY - mY - \gamma Y$.

The parameters are given as:

$r = 0.025$: intrinsic growth rate for prey,

$K = 1$: carrying capacity for prey,

$a = 0.08$: predator-related mortality parameter for prey,

$b = 0.05$: predator/prey uptake parameter for predator,

$m = 0.01$: natural mortality for predator,

$\gamma = 0.005$: slope for linear predator harvest control rule, and

$\theta = 0.005$: slope for linear prey harvest control rule

The predator with no economic value (unharvested) is designed for the economic program as:

$$W = \text{harv.prey}(p.\text{prey} - c.\text{prey}/X)\theta X + \text{harv.pred} * (p.\text{pred} - c.\text{pred}/Y)\gamma Y.$$

The paramters are:

$p.\text{pred} = 0$: price per unit harvest of predator,

$p.\text{prey} = 25$: price per unit harvest of prey,

$c.\text{prey} = 0.1p.\text{prey}$: cost /per unit of prey effort in Schaefer model (really c/q with $q=1$), and

$c.\text{pred} = c_p.\text{prey}$: cost per unit of predator effort in Schaefer model (really c/q with $q=1$).

Usage

```
## Load dataset
data("lvdata")
```

Format

lvsimdata.time: a data for time simulation (101 ODE solution)

- tseq time sequence from 0 to 100
- xs prey stock
- ys predator stock

See Also[LV](#), [vsim](#)

paprox

*Calculating P-approximation coefficients***Description**

The function provides the P-approximation coefficients of the defined Chebyshev polynomials in `aproxdef`. For now, only unidimensional case is developed.

Usage

```
paprox(aproxspace, stock, sdot, dsdotds, dwds)
```

Arguments

<code>aproxspace</code>	An approximation space defined by <code>aproxdef</code> function
<code>stock</code>	An array of stock, s
<code>sdot</code>	An array of ds/dt , $\dot{s} = \frac{ds}{dt}$
<code>dsdotds</code>	An array of $d(sdot)/ds$, $\frac{d\dot{s}}{ds}$
<code>dwds</code>	An array of dw/ds , $\frac{dW}{ds}$

Details

The P-approximation is finding the shadow price of a stock, p from the relation:

$$p(s) = \frac{W_s(s) + \dot{p}(s)}{\delta - \dot{s}_s},$$

where $W_s = \frac{dW}{ds}$, $\dot{p}(s) = \frac{dp}{ds}$, $\dot{s}_s = \frac{d\dot{s}}{ds}$, and δ is the given discount rate.

Consider approximation $p(s) = \mu(s)\beta$, $\mu(s)$ is Chebyshev polynomials and β is their coefficients. Then, $\dot{p} = \text{diag}(\dot{s})\mu_s(s)\beta$ by the orthogonality of Chebyshev basis. Adopting the properties above, we can get the unknown coefficient vector β from:

$$\mu\beta = \text{diag}(\delta - \dot{s}_s)^{-1} (W_s + \text{diag}(\dot{s})\mu_s\beta), \text{ and thus,}$$

$$\beta = (\text{diag}(\delta - \dot{s}_s)\mu - \text{diag}(\dot{s})\mu_s)^{-1} W_s.$$

In a case of over-determined (more nodes than approximation degrees),

$$\left((\text{diag}(\delta - \dot{s}_s)\mu - \text{diag}(\dot{s})\mu_s)^T (\text{diag}(\delta - \dot{s}_s)\mu - \text{diag}(\dot{s})\mu_s) \right)^{-1} (\text{diag}(\delta - \dot{s}_s)\mu - \text{diag}(\dot{s})\mu_s)^T W_s$$

For more details see [Fenichel et al. \(2016\)](#).

Value

A list of approximation results: deg, lb, ub, delta, and coefficients. Use results\$item (or results[["item"]]) to import each result item.

degree	degree of Chebyshev polynomial
lowerB	lower bound of Chebyshev nodes
upperB	upper bound of Chebyshev nodes
delta	discount rate
coefficient	Chebyshev polynomial coefficients

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

Fenichel, Eli P., Joshua K. Abbott, Jude Bayham, Whitney Boone, Erin M. K. Haacker, and Lisa Pfeiffer. (2016) "**Measuring the Value of Groundwater and Other Forms of Natural Capital.**" *Proceedings of the National Academy of Sciences* .113:2382-2387.

See Also

[aproxdef](#), [psim](#)

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("GOM")
nodes <- chebnodegen(param$nodes,param$lowerK,param$upperK)
simuDataP <- cbind(nodes,sdot(nodes,param),
                   dsdotds(nodes,param),dwds(nodes,param))
Aspace <- aproxdef(param$order,param$lowerK,param$upperK,param$delta)
pC <- paprox(Aspace,simuDataP[,1],simuDataP[,2],
             simuDataP[,3],simuDataP[,4])
```

param	<i>the parameter vector adopted in GOM dataset</i>
-------	--

Description

The GOM provides data to replicate the Gulf of Mexico Reef Fish example in [Fenichel and Abbott \(2014\)](#).

Usage

```
## Load dataset
data("GOM")
```

Format

param: a data.frame of parameters

- r intrinsic growth rate (=0.3847)
- k carrying capacity (=359016000)
- q catchability coefficient (=0.00031729344157311126)
- price price (=2.70)
- cost cost (=153.0)
- alpha technology parameter (=0.5436459179063678)
- gamma pre-ITQ management parameter (=0.7882)
- y system equivalence parameter (=0.15745573410462155)
- delta discount rate (=0.02)
- order Chebyshev polynomial order (=50)
- upperK upper bound of Chebyshev polynomial nodes (=k)
- lowerK lower bound of Chebyshev polynomial nodes ($=5 \cdot 10^6$)
- nodes the number of Chebyshev polynomial nodes (=50)

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

pdotapprox

Calculating Pdot-approximation coefficients

Description

The function provides the Pdot-approximation coefficients of the defined Chebyshev polynomials in aproxdef. For now, only unidimensional case is developed.

Usage

```
pdotapprox(aproxspace, stock, sdot, dsdotds, dsdotdss, dwds, dwdss)
```

Arguments

aproxspace	An approximation space defined by aproxdef function
stock	An array of stock, s
sdot	An array of ds/dt , $\dot{s} = \frac{ds}{dt}$
dsdotds	An array of $d(\text{sdot})/ds$, $\frac{d\dot{s}}{ds}$
dsdotdss	An array of $d/ds(d(\text{sdot})/ds)$, $\frac{d}{ds} \left(\frac{d\dot{s}}{ds} \right)$
dwdss	An array of dw/ds , $\frac{dW}{ds}$
dwdss	An array of $d/ds(dw/ds)$, $\frac{d}{ds} \left(\frac{dW}{ds} \right)$

Details

The Pdot-approximation is finding the shadow price of a stock, p from the relation:

$$p(s) = \frac{W_s(s) + \dot{p}(s)}{\delta - \dot{s}_s},$$

where $W_s = \frac{dW}{ds}$, $\dot{p}(s) = \frac{dp}{ds}$, $\dot{s}_s = \frac{d\dot{s}}{ds}$, and δ is the given discount rate.

In order to operationalize this approach, we take the time derivative of this expression:

$$\dot{p} = \frac{((W_{ss}\dot{s} + \ddot{p})(\delta - \dot{s}_s) + (W_s + \dot{p})(\dot{s}_{ss}\dot{s}))}{(\delta - \dot{s}_s)^2}$$

Consider approximation $\dot{p}(s) = \mu(s)\beta$, $\mu(s)$ is Chebyshev polynomials and β is their coefficients. Then, $\ddot{p} = \frac{d\dot{p}}{ds} \frac{ds}{dt} = \text{diag}(\dot{s})\mu_s(s)\beta$ by the orthogonality of Chebyshev basis. Adopting the properties above, we can get the unknown coefficient vector β from:

$$\mu\beta = \text{diag}(\delta - \dot{s}_s)^{-2} [(W_{ss}\dot{s} + \text{diag}(\dot{s})\mu_s\beta)(\delta - \dot{s}_s) + \text{diag}(\dot{s}_{ss}\dot{s})(W_s + \mu\beta)], \text{ and}$$

$$\beta = \left[\text{diag}(\delta - \dot{s}_s)^2 \mu - \text{diag}(\dot{s}(\delta - \dot{s}_s)) \mu_s - \text{diag}(\dot{s}_{ss}\dot{s}) \mu \right]^{-1} (W_{ss}\dot{s}(\delta - \dot{s}_s) + W_s\dot{s}_{ss}\dot{s}).$$

If we suppose $A = \left[\text{diag}(\delta - \dot{s}_s)^2 \mu - \text{diag}(\dot{s}(\delta - \dot{s}_s)) \mu_s - \text{diag}(\dot{s}_{ss}\dot{s}) \mu \right]$ and $B = (W_{ss}\dot{s}(\delta - \dot{s}_s) + W_s\dot{s}_{ss}\dot{s})$, then over-determined case can be calculated:

$$\beta = (A^T A)^{-1} A^T B.$$

For more details see [Fenichel and Abbott \(2014\)](#).

Value

A list of approximation results: deg, lb, ub, delta, and coefficients. Use `results$item` (or `results[["item"]]`) to import each result item.

degree	degree of Chebyshev polynomial
lowerB	lower bound of Chebyshev nodes
upperB	upper bound of Chebyshev nodes
delta	discount rate
coefficient	Chebyshev polynomial coefficients

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[aproxdef](#), [pdotsim](#)

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("GOM")
nodes <- chebnodegen(param$nodes,param$lowerK,param$upperK)
simuDataPdot <- cbind(nodes,sdot(nodes,param),
                      dsdotds(nodes,param),dsdotdss(nodes,param),
                      dwds(nodes,param),dwdss(nodes,param))
Aspace <- aproxdef(param$order,param$lowerK,param$upperK,param$delta)
pdotC <- pdotaprox(Aspace,simuDataPdot[,1],simuDataPdot[,2],
                  simuDataPdot[,3],simuDataPdot[,4],
                  simuDataPdot[,5],simuDataPdot[,6])
```

pdotsim

Simulation of Pdot-approximation

Description

The function provides the Pdot-approximation simulation.

Usage

```
pdotsim(pdotcoeff, stock, sdot, dsdotds, wval, dwds)
```

Arguments

pdotcoeff	An approximation result from pdotaprox function
stock	An array of stock
sdot	An array of ds/dt , $\dot{s} = \frac{ds}{dt}$
dsdotds	An array of $d(sdot)/ds$, $\frac{d\dot{s}}{ds}$
wval	An array of W -value
dwds	An array of dw/ds , $\frac{dW}{ds}$


```
# Shadow Price
plotgen(GOMSimPdot, xlabel="Stock size, s", ylabel="Shadow price")

# Value function and profit
plotgen(GOMSimPdot, ftype="vw",
        xlabel="Stock size, s",
        ylabel=c("Value Function", "Profit"))
```

plotgen

*Plot Generator for Shadow Price or Value Function***Description**

The function draws shadowp or vfun-w plot from the simulation results of vsim, psim, or pdotsim.

Usage

```
plotgen(simres, ftype = NULL, whichs = NULL, tvar = NULL, xlabel = NULL,
        ylabel = NULL)
```

Arguments

simres	A simulation results from vsim, psim, or pdotsim
ftype	Plot type (ftype=NULL (default) or ftype="p" for shadow price; ftype="vw" for vfun-w plot)
whichs	A positive integer for indicating a specific stock for multi-stock cases (ftype=NULL (default) or 1 <= whichs <= the number of stocks)
tvar	An array of time variable if simulation result is a time-base simulation
xlabel	A character for x-label of a plot (xlabel=NULL (default); "Stock" or "Time")
ylabel	An array of characters for y-label of a plot (ylabel=NULL (default); "Shadow Price", "Value Function" or "W-value")

Details

This function provides an one-dimensional plot for "shadow price-stock", "shadow price-time", "Value function-stock", "Value function-time", "Value function-stock-W value", or "Value function-time-W value" depending on input arguments.

Value

A plot of approximation results: shadow (accounting) prices, inclusive wealth, and Value function

See Also

[vsim](#), [psim](#), [pdotsim](#)

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("GOM")
nodes <- chebnodegen(param$nodes,param$lowerK,param$upperK)
simuDataP <- cbind(nodes,sdot(nodes,param),
                  dsdotds(nodes,param),dwds(nodes,param))
Aspace <- aproxdef(param$order,param$lowerK,param$upperK,param$delta)

# p-approximation
pC <- paprox(Aspace,simuDataP[,1],simuDataP[,2],
            simuDataP[,3],simuDataP[,4])

# Without providing W-value
GOMSimP <- psim(pC,simuDataP[,1])
# With W-value
GOMSimP2 <- psim(pC,simuDataP[,1],profit(nodes,param),simuDataP[,2])

# Shadow price-Stock plot
plotgen(GOMSimP)
plotgen(GOMSimP,ftype="p")
plotgen(GOMSimP,xlabel="Stock Size, S", ylabel="Shadow Price (USD/Kg)")

# Value-Stock-W plot
plotgen(GOMSimP2,ftype="vw")
plotgen(GOMSimP2,ftype="vw",xlabel="Stock Size, S", ylabel="Value Function")
plotgen(GOMSimP2,ftype="vw",xlabel="Stock Size, S", ylabel="Value Function")

## 2-D Prey-Predator example
data("lvdata")
aproxdeg <- c(20,20)
lower <- c(0.1,0.1)
upper <- c(1.5,1.5)
delta <- 0.03
lvspace <- aproxdef(proxdeg,lower,upper,delta)
lvaprox <- vaprox(lvspace,lvaproxdata)
lvsim <- vsim(lvaprox,lvsimdata.time[,2:3])

# Shadow price-Stock plot
plotgen(lvsim)
plotgen(lvsim,ftype="p")
plotgen(lvsim,whichs=2,xlabel="Stock Size, S",ylabel="Shadow Price (USD/Kg)")

# Shadow price-time plot
plotgen(lvsim,whichs=2,tvar=lvsimdata.time[,1])

# Value Function-Stock plot
plotgen(lvsim,ftype="vw")
plotgen(lvsim,ftype="vw",whichs=2,
        xlabel="Stock Size, S",ylabel="Shadow Price (USD/Kg)")

# Value Function-time plot
plotgen(lvsim,ftype="vw",tvar=lvsimdata.time[,1])
```



```
plotgen(lvsim,ftype="vw",whichs=2,tvar=lvsimdata.time[,1],
        xlabel="Stock Size, S",ylabel="Shadow Price (USD/Kg)")
```

profit	<i>profit function in GOM dataset</i>
--------	---------------------------------------

Description

profit (w) function in the reef-fishy example of GOM dataset (Fenichel and Abbott, 2014).

Usage

```
profit(s,Z)
```

Arguments

s	stock
Z	parameter vector

Details

This function is adopted in GOM dataset.

Value

```
profit
```

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

psim

*Simulation of P-approximation***Description**

The function provides the P-approximation simulation.

Usage

```
psim(pcoeff, stock, wval = NULL, sdot = NULL)
```

Arguments

pcoeff	An approximation result from paprox function
stock	An array of stock variable
wval	(Optional for vfun) An array of W -value (need sdot simultaneously)
sdot	(Optional for vfun) An array of ds/dt , $\dot{s} = \frac{ds}{dt}$ (need W simultaneously)

Details

Let $\hat{\beta}$ be the vector of approximation coefficients from the results of paprox function. The estimated shadow price (accounting) price of stock over the given approximation interval of $s \in [a, b]$, $\hat{p}$ can be calculated as:

$$\hat{p} = \mu(s)\hat{\beta}.$$

The estimated value function is:

$$\hat{V} = \frac{1}{\delta} (W + \hat{p}\dot{s}).$$

For more details see [Fenichel and Abbott \(2014\)](#) and [Fenichel et al. \(2016\)](#).

Value

A list of approximation results: shadow (accounting) prices, inclusive wealth, value function, stock, and W values. Use `results$item` (or `results[["item"]]`) to import each result item.

shadowp	Shadow price
vfun	Value function
stock	Stock
wval	W-value if wval is provided

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.
Fenichel, Eli P., Joshua K. Abbott, Jude Bayham, Whitney Boone, Erin M. K. Haacker, and Lisa Pfeiffer. (2016) "**Measuring the Value of Groundwater and Other Forms of Natural Capital.**" *Proceedings of the National Academy of Sciences* .113:2382-2387.

See Also

[aproxdef](#), [paprox](#)

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("GOM")
nodes <- chebnodegen(param$nodes,param$lowerK,param$upperK)
simuDataP <- cbind(nodes,sdot(nodes,param),
                  dsdotds(nodes,param),dwds(nodes,param))
Aspace <- aproxdef(param$order,param$lowerK,param$upperK,param$delta)
pC <- paprox(Aspace,simuDataP[,1],simuDataP[,2],
            simuDataP[,3],simuDataP[,4])
GOMSimP <- psim(pC,simuDataP[,1],profit(nodes,param),simuDataP[,2])

# Shadow Price
plotgen(GOMSimP, xlabel="Stock size, s", ylabel="Shadow price")

# Value function and profit
plotgen(GOMSimP,ftype="vw",
        xlabel="Stock size, s",
        ylabel=c("Value Function","Profit"))
```

sdot	<i>growth function of GOM dataset</i>
------	---------------------------------------

Description

The function calculates the growth rate in the reef-fishy example of GOM dataset (Fenichel and Abbott, 2014).

Usage

```
sdot(s,Z)
```

Arguments

- | | |
|---|------------------|
| s | stock |
| Z | parameter vector |

Details

This function is adopted in GOM dataset.

Value

growth rate

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

See Also

[GOM](#)

unigrids

Generating unifrom grids

Description

This function generates a grid of multi-dimensional uniform grids.

Usage

```
unigrids(nnodes, lb, ub, rtype = NULL)
```

Arguments

nnodes	An array of numbers of nodes
lb	An array of lower bounds
ub	An array of upper bounds
rtype	A type of results; default is NULL that returns a list class; if rtype = list, returns a list class; if rtype = grid, returns a matrix class.

Details

For the i -th dimension of $i = 1, 2, \dots, d$, suppose a polynomial approximant s_i over a bounded interval $[a_i, b_i]$ is defined by evenly gridded nodes. Then, a d -dimensional uniform grids can be defined as:

$$\mathbf{S} = \{(s_1, s_2, \dots, s_d) | a_i \leq s_i \leq b_i, i = 1, 2, \dots, d\}.$$

This is all combinations of s_i . Two types of results are provided. 'rtype = list' provides a list of d dimensions wheras 'rtype = grids' creates a $\left(\prod_{i=1}^d n_i\right) \times d$ matrix.

Value

A list with d elements of Chebyshev nodes or a $\left(\prod_{i=1}^d n_i\right) \times d$ matrix of uniform grids

Examples

```
## Uniform grids with two-dimension
unigrids(c(5,3), c(1,1), c(2,3))
## Returns the same results
unigrids(c(5,3), c(1,1), c(2,3), rtype='list')
## Returns a matrix grids with the same domain
unigrids(c(5,3), c(1,1), c(2,3), rtype='grid')
## Uniform grid with one-dimension
unigrids(5,1,2)
## Uniform grids with three stock
unigrids(c(3,4,5),c(1,1,1),c(2,3,4),rtype='grid')
```

vaprox

*Calculating V-approximation coefficients***Description**

The function provides the V-approximation coefficients of the defined Chebyshev polynomials in aproxdef.

Usage

```
vaprox(aproxspace, sdata)
```

Arguments

aproxspace	An approximation space defined by aproxdef function
sdata	A data.frame or matrix of [stock,sdot,benefit]=[$\mathbf{S}, \dot{\mathbf{S}}, \mathbf{W}$]

Details

The V-approximation is finding the shadow price of i -th stock, p_i for $i = 1, \dots, d$ from the relation:

$$\delta V = W(\mathbf{S}) + p_1 \dot{s}_1 + p_2 \dot{s}_2 + \dots + p_d \dot{s}_d,$$

where δ is the given discount rate, V is the intertemporal welfare function, $\mathbf{S} = (s_1, s_2, \dots, s_d)$ is a vector of stocks, $W(\mathbf{S})$ is the net benefits accruing to society, and $\dot{s}_i$ is the growth of stock s_i . By the definition of the shadow price, we know:

$$p_i = \frac{\partial V}{\partial s_i}.$$

Consider approximation $V(\mathbf{S}) = \mu(\mathbf{S})\beta$, $\mu(\mathbf{S})$ is Chebyshev polynomials and β is their coefficients. Then, $p_i = \mu_{s_i}(\mathbf{S})\beta$ by the orthogonality of Chebyshev basis. Adopting the properties above, we can get the unknown coefficient vector β from:

$$\delta\mu(\mathbf{S})\beta = W(\mathbf{S}) + \sum_{i=1}^d \text{diag}(\dot{s}_i)\mu_{s_i}(\mathbf{S})\beta, \text{ and thus,}$$

$$\beta = \left(\delta\mu(\mathbf{S}) - \sum_{i=1}^d \text{diag}(\dot{s}_i)\mu_{s_i}(\mathbf{S}) \right)^{-1} W(\mathbf{S}).$$

In a case of over-determined (more nodes than approximation degrees),

$$\beta = \left(\left(\delta\mu(\mathbf{S}) - \text{diag}(\dot{s}_i) \sum_{i=1}^d \mu_{s_i}(\mathbf{S}) \right)^T \left(\delta\mu(\mathbf{S}) - \sum_{i=1}^d \text{diag}(\dot{s}_i)\mu_{s_i}(\mathbf{S}) \right) \right)^{-1}$$

$$\times \left(\delta\mu(\mathbf{S}) - \sum_{i=1}^d \text{diag}(\dot{s}_i)\mu_{s_i}(\mathbf{S}) \right)^T W(\mathbf{S}).$$

For more details see Fenichel and Abbott (2014), Fenichel et al. (2016), and Yun et al. (2017).

Value

A list of approximation results: deg, lb, ub, delta, and coefficients. Use `results$item` (or `results[["item"]]`) to import each result item.

degree	degree of Chebyshev polynomial
lowerB	lower bound of Chebyshev nodes
upperB	upper bound of Chebyshev nodes
delta	discount rate
coefficient	Chebyshev polynomial coefficients

References

- Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.
- Fenichel, Eli P., Joshua K. Abbott, Jude Bayham, Whitney Boone, Erin M. K. Haacker, and Lisa Pfeiffer. (2016) "**Measuring the Value of Groundwater and Other Forms of Natural Capital.**" *Proceedings of the National Academy of Sciences*. 113:2382-2387.
- Yun, Seong Do, Barbara Hutniczak, Joshua K. Abbott, and Eli P. Fenichel. (2017) "Ecosystem Based Management and the Wealth of Ecosystems" *Proceedings of the National Academy of Sciences*. (forthcoming).

See Also

[aproxdef](#), [vsim](#)

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("GOM")
nodes <- chebnodegen(param$nodes,param$lowerK,param$upperK)
simuDataV <- cbind(nodes,sdot(nodes,param),profit(nodes,param))
Aspace <- aproxdef(param$order,param$lowerK,param$upperK,param$delta)
vC <- vaprox(Aspace,simuDataV)

## 2-D Prey-Predator example
data("lvdata")
aproxdeg <- c(20,20)
lower <- c(0.1,0.1)
upper <- c(1.5,1.5)
delta <- 0.03
lvspace <- aproxdef(aproxdeg,lower,upper,delta)
vaprox <- vaprox(lvspace,lvaproxdata)
```

vsim

Simulation of V-approximation

Description

The function provides the V-approximation simulation by adopting the results of vaprox. Available for multiple stock problems.

Usage

```
vsim(vcoeff, adata, wval = NULL)
```

Arguments

vcoeff	An approximation result from varpox function
adata	A data.frame or matrix of [stock]=[S]
wval	(Optional for plotgen) An array of W -value

Details

Let $\hat{\beta}$ be the approximation coefficient from the results of vaprox function. The estimated shadow (accounting) price of i -th stock over the given approximation intervals of $s_i \in [a_i, b_i]$, $\hat{p}_i$ can be calculated as:

$$\hat{p}_i = \mu(\mathbf{S})\hat{\beta} \text{ where } \mu(\mathbf{S}) \text{ Chebyshev polynomial basis.}$$

The value function is:

$$\hat{V} = \delta\mu(\mathbf{S})\hat{\beta}.$$

For more details see Fenichel and Abbott (2014), Fenichel et al. (2016a), Fenichel et al. (2016b), and Yun et al. (2017).

Value

A list of simulation results: shadow (accounting) prices, inclusive wealth, Value function, stock, and W values. Use `results$item` (or `results[["item"]]`) to import each result item.

<code>shadowp</code>	Shadow price
<code>iweach</code>	Inclusive wealth for each stock for multi-stock case
<code>vfun</code>	Value function
<code>stock</code>	Stock
<code>wval</code>	W-value if <code>wval</code> is provided

References

Fenichel, Eli P. and Joshua K. Abbott. (2014) "**Natural Capital: From Metaphor to Measurement.**" *Journal of the Association of Environmental Economists*. 1(1/2):1-27.

Fenichel, Eli P., Joshua K. Abbott, Jude Bayham, Whitney Boone, Erin M. K. Haacker, and Lisa Pfeiffer. (2016a) "**Measuring the Value of Groundwater and Other Forms of Natural Capital.**" *Proceedings of the National Academy of Sciences*. 113:2382-2387.

Fenichel, Eli P., Simon A. Levin, Bonnie McCay, Kevin St. Martin, Joshua K. Abbott, and Malin L. Pinsky. (2016b) "**Wealth Reallocation and Sustainability under Climate Change.**" *Nature Climate change*. 6:237-244.

Yun, Seong Do, Barbara Hutniczak, Joshua K. Abbott, and Eli P. Fenichel. (2017) "Ecosystem Based Management and the Wealth of Ecosystems" *Proceedings of the National Academy of Sciences*. (forthcoming).

See Also

[aproxdef](#), [vsim](#)

Examples

```
## 1-D Reef-fish example: see Fenichel and Abbott (2014)
data("GOM")
nodes <- chebnodegen(param$nodes,param$lowerK,param$upperK)
simuDataV <- cbind(nodes,sdot(nodes,param),profit(nodes,param))
Aspace <- aproxdef(param$order,param$lowerK,param$upperK,param$delta)
vC <- vaprox(Aspace,simuDataV)
# Note vcol function requires a data.frame or matrix!
GOMSimV <- vsim(vC,as.matrix(simuDataV[,1],ncol=1),profit(nodes,param))

# plot shadow (accounting) price: Figure 4 in Fenichel and Abbott (2014)
plotgen(GOMSimV, xlabel="Stock size, s", ylabel="Shadow price")

## 2-D Prey-Predator example
data("lvdata")
aproxdeg <- c(20,20)
lower <- c(0.1,0.1)
upper <- c(1.5,1.5)
delta <- 0.03
lvspace <- aproxdef(aproxdeg,lower,upper,delta)
```



```

lvaprox <- vaprox(lvspace,lvaproxdata)
lvsim <- vsim(lvaprox,lvsimdata.time[,2:3])

# plot Biomass
plot(lvsimdata.time[,1], lvsimdata.time[,2], type='l', lwd=2, col="blue",
      xlab="Time",
      ylab="Biomass")
lines(lvsimdata.time[,1], lvsimdata.time[,3], lwd=2, col="red")
legend("topright", c("Prey", "Predator"), col=c("blue", "red"),
      lty=c(1,1), lwd=c(2,2), bty="n")

# plot shadow (accounting) prices
plot(lvsimdata.time[,1],lvsim[["shadowp"]][,1],type='l', lwd=2, col="blue",
      ylim = c(-5,7),
      xlab="Time",
      ylab="Shadow price")
lines(lvsimdata.time[,1],lvsim[["shadowp"]][,2], lwd=2, col="red")
legend("topright", c("Prey", "Predator"), col=c("blue", "red"),
      lty=c(1,1), lwd=c(2,2), bty="n")

# plot inclusive weath and value function
plot(lvsimdata.time[,1],lvsim[["iw"]],type='l', lwd=2, col="blue",
      ylim = c(-0.5,1.2),
      xlab="Time",
      ylab="Inclusive Wealth / Value Function ($)")
lines(lvsimdata.time[,1],lvsim[["vfun"]], lwd=2, col="red")
legend("topright", c("Inclusive Wealth", "Value Function"),
      col=c("blue", "red"), lty=c(1,1), lwd=c(2,2), bty="n")

```

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