

$$\begin{aligned}
f(z) &= \frac{1}{2\pi i} \oint_C \sum_{n=0}^{\infty} \frac{(z-a)^n}{(z_0-a)^{n+1}} f(z_0) \, dz_0 \\
&= \sum_{n=0}^{\infty} \frac{(z-a)^n}{n!} \frac{n!}{2\pi i} \oint_C \frac{f(z_0)}{(z_0-a)^{n+1}} \, dz_0 \\
&= \sum_{n=0}^{\infty} \frac{(z-a)^n}{n!} f^{(n)}(a)
\end{aligned}$$