

# Package ‘qspray’

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**Type** Package

**Title** Multivariate Polynomials with Rational Coefficients

**Version** 3.0.0

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**Description** Symbolic calculation and evaluation of multivariate polynomials with rational coefficients. This package is strongly inspired by the 'spray' package. It provides a function to compute Gröbner bases (reference <doi:10.1007/978-3-319-16721-3>). The header file of the C++ code can be used by other packages. It provides the templated class 'Qspray' that can be used to represent and to deal with multivariate polynomials with another type of coefficients.

**License** GPL-3

**URL** <https://github.com/stla/qspray>

**BugReports** <https://github.com/stla/qspray/issues>

**Imports** DescTools, gmp, methods, partitions, purrr, RationalMatrix, Rcpp (>= 1.0.9), Ryacas, utils

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---

as.function.qspray      *Multivariate polynomial as function*

---

## Description

Coerces a qspray polynomial into a function.

## Usage

```
## S3 method for class 'qspray'
as.function(x, N = FALSE, ...)
```

## Arguments

x	object of class qspray
N	Boolean, whether the function must numerically approximate the result
...	ignored

## Value

A function having the same variables as the polynomial. If N=FALSE, it returns a string. If N=TRUE, it returns a number if the result does not contain any variable, otherwise it returns a R expression.

## Examples

```
library(qspray)
P <- (qlone(1) + "1/2"*qlone(2))^2 + 5
f <- as.function(P)
g <- as.function(P, N = TRUE)
f(2, "3/7")
g(2, "3/7")
f("x", "y")
g("x", "y")
# the evaluation is performed by (R)yacas and complex numbers are
# allowed; the imaginary unit is denoted by `I`
f("2 + 2*I", "Sqrt(2)")
g("2 + 2*I", "Sqrt(2)")
```

---

 as.qspray

*Coercion to a 'qspray' object*


---

### Description

Coercion to a 'qspray' object

### Usage

```
## S4 method for signature 'character'
as.qspray(x)
```

```
## S4 method for signature 'qspray'
as.qspray(x)
```

```
## S4 method for signature 'numeric'
as.qspray(x)
```

```
## S4 method for signature 'bigz'
as.qspray(x)
```

```
## S4 method for signature 'bigq'
as.qspray(x)
```

### Arguments

x a qspray object or an object yielding a quoted integer or a quoted fraction after an application of `as.character`, e.g. a `bigq` number

### Value

A qspray object.

### Examples

```
as.qspray(2)
as.qspray("1/3")
```

---

 changeVariables

*Change of variables in a 'qspray' polynomial*


---

### Description

Replaces the variables of a qspray polynomial with some qspray polynomials. E.g. you have a polynomial  $P(x, y)$  and you want the polynomial  $P(x^2, x+y+1)$ . This is an alias of [composeQspray](#).

**Usage**

```
## S4 method for signature 'qspray,list'
changeVariables(x, listOfQsprays)
```

**Arguments**

`x` a qspray polynomial

`listOfQsprays` a list containing at least `n` qspray objects, or objects coercable to qspray objects, where `n` is the number of variables of the polynomial given in the `x` argument

**Value**

The qspray polynomial obtained by replacing the variables of the polynomial given in the `x` argument with the polynomials given in the `listOfQsprays` argument.

**Examples**

```
library(qspray)
f <- function(x, y) x*y/2 + 4*y
x <- qlone(1)
y <- qlone(2)
P <- f(x, y)
X <- x^2
Y <- x + y + 1
changeVariables(P, list(X, Y)) == f(X, Y) # should be TRUE
```

---

characteristicPolynomial

*Characteristic polynomial*

---

**Description**

Characteristic polynomial of a matrix.

**Usage**

```
characteristicPolynomial(A)
```

**Arguments**

`A` a square matrix with numeric, character, or bigq entries

**Value**

A univariate qspray polynomial.

**Examples**

```

set.seed(666)
A <- matrix(rpois(9L, 10), nrow = 3, ncol = 3)
( P <- characteristicPolynomial(A) )
# check the roots are the eigen values:
f <- as.function(P, N = TRUE)
sapply(eigen(A)$values, f) # approx c(0, 0, 0)

```

---

collinearQsprays      *Whether two 'qspray' polynomials are collinear*

---

**Description**

Checks whether the polynomials represented by two qspray objects are collinear, that is, whether they are equal up to a scalar factor.

**Usage**

```
collinearQsprays(qspray1, qspray2)
```

**Arguments**

```
qspray1, qspray2
                two qspray objects
```

**Value**

A Boolean value.

**Examples**

```

library(qspray)
qspray1 <- qsprayMaker(string = "1/2 x^(1, 1) + 4 x^(0, 2) + 5")
qspray2 <- "4/7" * qspray1
collinearQsprays(qspray1, qspray2)

```

---

compactSymmetricQspray      *Compact symmetric qspray*

---

**Description**

Prints a symmetric qspray polynomial as a linear combination of the monomial symmetric polynomials.

**Usage**

```
## S4 method for signature 'qspray,logical'
compactSymmetricQspray(qspray, check)

## S4 method for signature 'qspray,ANY'
compactSymmetricQspray(qspray)
```

**Arguments**

qspray            a qspray object, which should correspond to a symmetric polynomial  
 check            Boolean, whether to check the symmetry (default TRUE)

**Value**

A character string.

**See Also**

[MSPcombination](#)

**Examples**

```
library(qspray)
( qspray <- PSFpoly(4, c(3, 1)) - ESFpoly(4, c(2, 2)) + 4L )
compactSymmetricQspray(qspray, check = TRUE)
```

---

composeQspray            *Compose 'qspray' polynomials*

---

**Description**

Substitutes the variables of a qspray polynomial with some qspray polynomials. E.g. you have a polynomial  $P(x, y)$  and you want the polynomial  $P(x^2, x + y + 1)$  (see example).

**Usage**

```
composeQspray(qspray, listOfQsprays)
```

**Arguments**

qspray            a qspray polynomial  
 listOfQsprays    a list containing at least n qspray polynomials where n is the number of variables of the polynomial given in the qspray argument

**Value**

The qspray polynomial obtained by composing the polynomial given in the qspray argument with the polynomials given in the listOfQsprays argument.

**Examples**

```
library(qspray)
x <- qlone(1)
y <- qlone(2)
P <- x*y/2 + 4*y
X <- x^2
Y <- x + y + 1
composeQspray(P, list(X, Y)) # this is P(x^2, x+y+1)
```

---

derivQspray	<i>Partial derivative</i>
-------------	---------------------------

---

**Description**

Partial derivative of a qspray polynomial.

**Usage**

```
derivQspray(qspray, i, derivative = 1)
```

**Arguments**

qspray	object of class qspray
i	integer, the dimension to differentiate with respect to, e.g. 1 to differentiate with respect to $x$
derivative	integer, how many times to differentiate

**Value**

A qspray object.

**Examples**

```
library(qspray)
x <- qlone(1)
y <- qlone(2)
qspray <- 2*x + 3*x*y
derivQspray(qspray, 2) # derivative w.r.t. y
```

---

dQspray                      *Partial differentiation*

---

**Description**

Partial differentiation of a qspray polynomial.

**Usage**

```
dQspray(qspray, orders)
```

**Arguments**

qspray	object of class qspray
orders	integer vector, the orders of the differentiation; e.g. $c(2, 0, 1)$ means that you differentiate two times with respect to $x$ , you do not differentiate with respect to $y$ , and you differentiate one time with respect to $z$

**Value**

A qspray object.

**Examples**

```
library(qspray)
x <- qlone(1)
y <- qlone(2)
qspray <- x + 2*y + 3*x*y
dQspray(qspray, c(1, 1))
derivQspray(derivQspray(qspray, 1), 2)
```

---

ESFpoly                      *Elementary symmetric polynomial*

---

**Description**

Returns an elementary symmetric function as a polynomial.

**Usage**

```
ESFpoly(m, lambda)
```

**Arguments**

m	integer, the number of variables
lambda	an integer partition, given as a vector of decreasing positive integers

**Value**

A qspray object.

**Examples**

```
library(qspray)
ESFpoly(3, c(3, 1))
```

---

evalQspray

*Evaluate a 'qspray' object*

---

**Description**

Evaluation of the multivariate polynomial represented by a qspray object.

**Usage**

```
evalQspray(qspray, values_re, values_im = NULL)
```

**Arguments**

qspray	a qspray object
values_re	vector of the real parts of the values; each element of <code>as.character(values_re)</code> must be a quoted integer or a quoted fraction
values_im	vector of the imaginary parts of the values; each element of <code>as.character(values_im)</code> must be a quoted integer or a quoted fraction

**Value**

A bigq number if `values_im=NULL`, a pair of bigq numbers otherwise: the real part and the imaginary part of the result.

**Examples**

```
x <- qlone(1); y <- qlone(2)
P <- 2*x + "1/2"*y
evalQspray(P, c("2", "5/2", "99999")) # "99999" will be ignored
```

---

getCoefficient	<i>Get a coefficient in a 'qspray' polynomial</i>
----------------	---

---

**Description**

Get the coefficient corresponding to the given sequence of exponents.

**Usage**

```
## S4 method for signature 'qspray,numeric'
getCoefficient(qspray, exponents)
```

**Arguments**

qspray	a qspray object
exponents	a vector of exponents

**Value**

The coefficient as a bigq number.

**Examples**

```
library(qspray)
x <- qlone(1)
y <- qlone(2)
p <- 4*x^2 + 3*y - 5
getCoefficient(p, 2)
getCoefficient(p, c(2, 0)) # same as getCoefficient(p, 2)
getCoefficient(p, c(0, 1)) # coefficient of y because y=x^0*y^1
getCoefficient(p, 0) # the constant term
getCoefficient(p, integer(0)) # the constant term
getCoefficient(p, 3) # there's no cubic term
```

---

getConstantTerm	<i>Get the constant term of a 'qspray' polynomial</i>
-----------------	---

---

**Description**

Get the constant term of a qspray polynomial.

**Usage**

```
## S4 method for signature 'qspray'
getConstantTerm(qspray)
```

**Arguments**

qspray            a qspray object

**Value**

A bigq number.

---

groebner

*Gröbner basis*

---

**Description**

Returns a Gröbner basis following Buchberger's algorithm using the lexicographical order.

**Usage**

```
groebner(G, minimal = TRUE, reduced = TRUE)
```

**Arguments**

G                    a list of qspray polynomials, the generators of the ideal  
 minimal            Boolean, whether to return a minimal basis  
 reduced            Boolean, whether to return the reduced basis

**Value**

A Gröbner basis of the ideal generated by G, given as a list of qspray polynomials.

**References**

Cox, Little & O'Shea. *Ideals, Varieties, and Algorithms. An Introduction to Computational Algebraic Geometry and Commutative Algebra*. Fourth edition, Springer 2015.

**Examples**

```
library(qspray)
f <- qsprayMaker(string = "x^(3) - 2 x^(1,1)")
g <- qsprayMaker(string = "x^(2,1) - 2 x^(0,2) + x^(1)")
groebner(list(f, g), FALSE, FALSE)
# other example
x <- qlone(1); y <- qlone(2); z <- qlone(3)
f1 <- x^2 + y + z^2 - 1
f2 <- x^2 + y + z - 1
f3 <- x + y^2 + z - 1
gb <- groebner(list(f1, f2, f3))
lapply(gb, prettyQspray, vars = c("x", "y", "z"))
```

---

HallInnerProduct	<i>Hall inner product</i>
------------------	---------------------------

---

**Description**

Hall inner product of two symmetric polynomials.

**Usage**

```
HallInnerProduct(qspray1, qspray2, alpha = 1)
```

**Arguments**

qspray1, qspray2	two symmetric qspray polynomials
alpha	parameter equal to 1 for the usual Hall inner product, otherwise this is the "Jack parameter", an integer or a bigq number

**Value**

A bigq number.

---

implicitization	<i>Implicitization with Gröbner bases</i>
-----------------	---

---

**Description**

Implicitization of a system of parametric equations (see examples).

**Usage**

```
implicitization(nvariables, parameters, equations, relations)
```

**Arguments**

nvariables	number of variables
parameters	character vector of the names of the parameters, or NULL if there's no parameter
equations	list of qspray polynomials representing the parametric equations
relations	list of qspray polynomials representing the relations between the variables and the parameters, or NULL if there is none

**Value**

A list of qspray polynomials.

**Examples**

```

library(qspray)
# ellipse example #####
# variables
cost <- qlone(1)
sint <- qlone(2)
# parameters
a <- qlone(3)
b <- qlone(4)
#
nvariables <- 2
parameters <- c("a", "b")
equations <- list(
  "x" = a * cost,
  "y" = b * sint
)
relations <- list(
  cost^2 + sint^2 - 1
)
#
eqs <- implicitization(nvariables, parameters, equations, relations)

```

---

integratePolynomialOnSimplex

*Integral of a multivariate polynomial over a simplex*


---

**Description**

Returns the exact value of the integral of a multivariate polynomial with rational coefficients over a simplex whose vertices have rational coordinates.

**Usage**

```
integratePolynomialOnSimplex(P, S)
```

**Arguments**

P	a qspray object
S	the simplex, a (n+1)xn matrix such that each entry of the matrix as . character(S) is a quoted integer or a quoted fraction

**Value**

A bigq number, the exact value of the integral.

**Examples**

```
library(qspray)
x <- qlone(1); y <- qlone(2)
P <- x/2 + x*y
S <- rbind(c("0", "0"), c("1", "0"), c("1", "1")) # a triangle
integratePolynomialOnSimplex(P, S)
```

---

isConstant	<i>Whether a 'qspray' polynomial is constant</i>
------------	--

---

**Description**

Checks whether a qspray object defines a constant polynomial.

**Usage**

```
## S4 method for signature 'qspray'
isConstant(x)
```

**Arguments**

x                    a qspray object

**Value**

A Boolean value.

---

isPolynomialOf	<i>Whether a 'qspray' is a polynomial of some given 'qsprays'</i>
----------------	---

---

**Description**

Checks whether a qspray polynomial can be written as a polynomial of some given qspray polynomials. If TRUE, this polynomial is returned.

**Usage**

```
isPolynomialOf(qspray, qsprays)
```

**Arguments**

qspray                a qspray object  
qsprays               a list of qspray objects

**Value**

A Boolean value indicating whether the polynomial defined by `qspray` can be written as a polynomial of the polynomials defined by the `qspray` objects given in the `qsprays` list. If this is TRUE, this polynomial is returned as an attribute named "polynomial".

**Examples**

```
library(qspray)
P <- function(X, Y) X^2*Y + 2*X + 3
x <- qlone(1); y <- qlone(2); z <- qlone(3)
q1 <- x + y
q2 <- x*z^2 + 4
qspray <- P(q1, q2)
( check <- isPolynomialOf(qspray, list(q1, q2)) )
POLYNOMIAL <- attr(check, "polynomial")
composeQspray(POLYNOMIAL, list(q1, q2)) == qspray # should be TRUE
```

---

isQone

*Whether a 'qspray' polynomial is the unit polynomial*


---

**Description**

Checks whether a `qspray` object defines the unit polynomial.

**Usage**

```
## S4 method for signature 'qspray'
isQone(qspray)
```

**Arguments**

`qspray`            a `qspray` object

**Value**

A Boolean value.

---

isQzero	<i>Whether a 'qspray' polynomial is null</i>
---------	--

---

**Description**

Checks whether a qspray object defines the zero polynomial.

**Usage**

```
## S4 method for signature 'qspray'  
isQzero(qspray)
```

**Arguments**

qspray            a qspray object

**Value**

A Boolean value.

---

isSymmetricQspray	<i>Check symmetry of a polynomial</i>
-------------------	---------------------------------------

---

**Description**

Check whether a qspray polynomial is symmetric.

**Usage**

```
isSymmetricQspray(qspray)
```

**Arguments**

qspray            a qspray polynomial

**Value**

A Boolean value indicating whether the polynomial defined by qspray is symmetric.

**See Also**

[MSPcombination](#), [compactSymmetricQspray](#)

**Examples**

```
e1 <- ESFpoly(3, 1)
e2 <- ESFpoly(3, 2)
e3 <- ESFpoly(3, 3)
q <- e1 + 2*e2 + 3*e3 + 4*e1*e3
isSymmetricQspray(q)
```

---

isUnivariate	<i>Whether a 'qspray' is univariate</i>
--------------	---

---

**Description**

Checks whether a qspray object defines a univariate polynomial.

**Usage**

```
## S4 method for signature 'qspray'
isUnivariate(x)
```

**Arguments**

x                    a qspray object

**Value**

A Boolean value.

**Note**

It is considered that a constant qspray is univariate, and that the qspray object  $y^2+1$  where  $y=q\text{clone}(2)$  is not univariate, although only one variable is present (see the note in the documentation of `numberOfVariables`).

---

MSFpoly	<i>Monomial symmetric function</i>
---------	------------------------------------

---

**Description**

Returns a monomial symmetric function as a polynomial.

**Usage**

```
MSFpoly(m, lambda)
```

**Arguments**

`m` integer, the number of variables  
`lambda` an integer partition, given as a vector of decreasing positive integers

**Value**

A `qspray` object.

**Examples**

```
library(qspray)
MSFpoly(3, c(3, 1))
```

---

MSPcombination	<i>Symmetric polynomial in terms of the monomial symmetric polynomials</i>
----------------	--

---

**Description**

Expression of a symmetric polynomial as a linear combination of the monomial symmetric polynomials.

**Usage**

```
MSPcombination(qspray, check = TRUE)
```

**Arguments**

`qspray` a `qspray` object defining a symmetric polynomial  
`check` Boolean, whether to check the symmetry

**Value**

A list defining the combination. Each element of this list is a list with two elements: `coeff`, a big number, and `lambda`, an integer partition; then this list corresponds to the term `coeff * MSFpoly(n, lambda)`, where `n` is the number of variables in the symmetric polynomial.

**Examples**

```
qspray <- PSFpoly(4, c(3, 1)) + ESFpoly(4, c(2, 2)) + 4L
MSPcombination(qspray)
```

---

numberOfTerms	<i>Number of terms in a 'qspray' polynomial</i>
---------------	---

---

**Description**

Number of terms in the polynomial defined by a qspray object.

**Usage**

```
## S4 method for signature 'qspray'
numberOfTerms(qspray)
```

**Arguments**

qspray            a qspray object

**Value**

An integer.

---

numberOfVariables	<i>Number of variables in a 'qspray' polynomial</i>
-------------------	---

---

**Description**

Number of variables involved in a qspray object.

**Usage**

```
## S4 method for signature 'qspray'
numberOfVariables(x)
```

**Arguments**

x                    a qspray object

**Value**

An integer.

**Note**

The number of variables in the qspray object  $y^2+1$  where  $y=q1one(2)$  is 2, not 1, although only one variable is present. Strictly speaking, the function returns the maximal integer  $d$  such that the variable  $q1one(d)$  occurs in the polynomial.

---

orderedQspray	<i>Ordered 'qspray'</i>
---------------	-------------------------

---

**Description**

Reorders the terms of a qspray object according to the lexicographic order of the powers. This function is rather used internally only but it is exported for internal usage in other packages.

**Usage**

```
orderedQspray(qspray)
```

**Arguments**

qspray            a qspray object

**Value**

A qspray object. It defines the same polynomial as the input qspray object but it is ordered.

**Examples**

```
qspray <- rQspray()
qspray == orderedQspray(qspray) # should be TRUE
```

---

permuteVariables	<i>Permute variables</i>
------------------	--------------------------

---

**Description**

Permute the variables of a qspray polynomial.

**Usage**

```
## S4 method for signature 'qspray,numeric'
permuteVariables(x, permutation)
```

**Arguments**

x                    a qspray object  
permutation        a permutation

**Value**

A qspray object.

**Examples**

```
library(qspray)
f <- function(x, y, z) {
  x^2 + 5*y + z - 1
}
x <- qlone(1)
y <- qlone(2)
z <- qlone(3)
P <- f(x, y, z)
permutation <- c(3, 1, 2)
Q <- permuteVariables(P, permutation)
Q == f(z, x, y) # should be TRUE
```

---

prettyQspray

*Pretty polynomial*


---

**Description**

Pretty form of a qspray polynomial.

**Usage**

```
prettyQspray(qspray, vars = NULL)
```

**Arguments**

qspray	a qspray object
vars	variable names; NULL for "x1", "x2", ...

**Value**

A character string.

**Examples**

```
library(qspray)
P <- (qlone(1) + "1/2"*qlone(2))^2 + 5
prettyP <- prettyQspray(P, vars = c("x", "y"))
prettyP
cat(Ryacas::yac_str(sprintf("PrettyForm(%s)", prettyP)))
Ryacas::yac_str(sprintf("TeXForm(%s)", prettyP))
```

---

PSFpoly	<i>Power sum polynomial</i>
---------	-----------------------------

---

**Description**

Returns a power sum function as a polynomial.

**Usage**

```
PSFpoly(m, lambda)
```

**Arguments**

m	integer, the number of variables
lambda	an integer partition, given as a vector of decreasing positive integers

**Value**

A qspray object.

**Examples**

```
library(qspray)
PSFpoly(3, c(3, 1))
```

---

PSPexpression	<i>Symmetric polynomial in terms of the power sum polynomials.</i>
---------------	--

---

**Description**

Expression of a symmetric qspray polynomial as a polynomial in the power sum polynomials.

**Usage**

```
PSPexpression(qspray)
```

**Arguments**

qspray	a symmetric qspray polynomial (an error is returned if it is not symmetric)
--------	---

**Value**

A qspray polynomial, say  $P$ , such that  $P(p_1, \dots, p_n)$  equals the input symmetric polynomial, where  $p_i$  is the  $i$ -th power sum polynomial (`PSFpoly(n, i)`).

**Note**

This function has been exported because it is used in the **jack** package.

---

qdivision	<i>Division of a qspray polynomial</i>
-----------	--

---

**Description**

Division of a qspray polynomial by a list of qspray polynomials. See the reference for the definition.

**Usage**

```
qdivision(qspray, divisors)
```

**Arguments**

qspray	the dividend, a qspray object
divisors	the divisors, a list of qspray objects

**Value**

The remainder of the division, a qspray object.

**References**

Michael Weiss, 2010. [Computing Gröbner Bases in Python with Buchberger's Algorithm.](#)

**Examples**

```
# a univariate example
library(qspray)
x <- qlone(1)
f <- x^4 - 4*x^3 + 4*x^2 - x # 0 and 1 are trivial roots
g <- x * (x - 1)
qdivision(f, list(g)) # should be zero
```

---

qlone	<i>Polynomial variable</i>
-------	----------------------------

---

**Description**

Creates a polynomial variable. Using this function is the main way to build qspray objects.

**Usage**

```
qlone(n)
```

**Arguments**

n	positive integer, the index of the variable
---	---

**Value**

A qspray object.

**Examples**

```
x <- qlone(1)
y <- qlone(2)
(x + y) * (x - y)
```

---

qone

*The unit 'qspray' polynomial*


---

**Description**

Returns the qspray polynomial identically equal to 1.

**Usage**

```
qone()
```

**Value**

A qspray object.

---

qspray-unary

*Unary operators for qspray objects*


---

**Description**

Unary operators for qspray objects.

**Usage**

```
## S4 method for signature 'qspray,missing'
e1 + e2

## S4 method for signature 'qspray,missing'
e1 - e2
```

**Arguments**

e1	object of class qspray
e2	nothing

**Value**

A qspray object.

---

qsprayDivision	<i>Division of two polynomials</i>
----------------	------------------------------------

---

**Description**

Division of two polynomials

**Usage**

```
qsprayDivision(qsprayA, qsprayB)
```

**Arguments**

qsprayA	a qspray object, the dividend
qsprayB	a qspray object, the divisor

**Value**

A list with two qspray objects, the quotient and the remainder.

**Examples**

```
library(qspray)
x <- qlone(1)
y <- qlone(2)
z <- qlone(3)
B <- x*y^2 + z*x^2 + 1
A <- B * (x^2*y^2*z^2 - 3) + x*y
divis <- qsprayDivision(A, B)
B * divis[["Q"]] + divis[["R"]] == A # should be TRUE
```

---

qsprayMaker	<i>Make a 'qspray' object</i>
-------------	-------------------------------

---

**Description**

Make a qspray object from a list of exponents and a vector of coefficients.

**Usage**

```
qsprayMaker(powers, coeffs, string = NULL)
```

**Arguments**

powers	list of positive integer vectors
coeffs	a vector such that each element of <code>as.character(coeffs)</code> is a quoted integer or a quoted fraction; it must have the same length as the powers list
string	if not NULL, this argument takes precedence over powers and coeffs; it must be a string representing a multivariate polynomial; see the example

**Value**

A qspray object.

**Examples**

```
powers <- list(c(1, 1), c(0, 2))
coeffs <- c("1/2", "4")
qsprayMaker(powers, coeffs)
qsprayMaker(string = "1/2 x^(1, 1) + 4 x^(0, 2)")
```

---

qspray\_from\_list      *(internal) Make a 'qspray' object from a list*

---

**Description**

This function is for internal usage. It is exported because it is also used for internal usage in others packages.

**Usage**

```
qspray_from_list(qspray_as_list)
```

**Arguments**

qspray\_as\_list list returned by the Rcpp function `returnQspray`

**Value**

A qspray object.

qzero                      *The null 'qspray' polynomial*

---

**Description**

Returns the qspray polynomial identically equal to 0.

**Usage**

```
qzero()
```

**Value**

A qspray object.

---

rQspray                    *Random 'qspray'*

---

**Description**

Generates a random qspray object.

**Usage**

```
rQspray()
```

**Value**

A qspray object with at most 4 terms and at most 3 variables.

---

showMonomialOld         *Print a monomial*

---

**Description**

Prints a monomial like "x^(1, 0, 2)". This way of showing a monomial was used by default in previous versions of this package.

**Usage**

```
showMonomialOld(x = "x")
```

**Arguments**

x                          a string, usually a letter such as "x" or "X", to denote the variable

**Value**

A function which takes as argument a sequence of exponents and which prints the corresponding monomial.

**See Also**

[showMonomialX1X2X3](#), [showMonomialXYZ](#), [showQspray](#), [showQsprayOption<-](#).

**Examples**

```
showMonomialOld("X")(c(1, 0, 2))
showMonomialOld("X")(NULL)
```

---

```
showMonomialX1X2X3      Print a monomial
```

---

**Description**

Prints a monomial in the style of "x1 . x3^2".

**Usage**

```
showMonomialX1X2X3(x = "x", collapse = ".")
```

**Arguments**

x                    a string, usually a letter such as "x" or "X", to denote the non-indexed variables  
collapse            a string to denote the symbol representing the multiplication, e.g. "\*" or ""

**Value**

A function which takes as argument a sequence of exponents and which prints the corresponding monomial.

**Note**

The function returned by this function can be used as the option "showMonomial" in the [showQsprayOption<-](#) function. But if you are happy with the default collapse argument, then you can equivalently set the "x" option instead, thereby typing less code. See the example.

**See Also**

[showQsprayX1X2X3](#), [showMonomialXYZ](#), [showQsprayOption<-](#).

**Examples**

```

showMonomialX1X2X3("X")(c(1, 0, 2))
showMonomialX1X2X3("X", collapse = "*")(c(1, 0, 2))
showMonomialX1X2X3("X")(c(1, 0, 2)) ==
  showMonomialXYZ(c("X1", "X2", "X3"))(c(1, 0, 2))
showMonomialX1X2X3()(NULL)
# setting a show option:
set.seed(3141)
( qspray <- rQspray() )
showQsprayOption(qspray, "showMonomial") <- showMonomialX1X2X3("X")
qspray
# this is equivalent to:
showQsprayOption(qspray, "showQspray") <- showQsprayX1X2X3("X")
# and also equivalent to:
showQsprayOption(qspray, "x") <- "X"

```

---

<code>showMonomialXYZ</code>	<i>Print a monomial</i>
------------------------------	-------------------------

---

**Description**

Prints a monomial like "x.z^2" if possible (see details).

**Usage**

```
showMonomialXYZ(letters = c("x", "y", "z"), collapse = ".")
```

**Arguments**

letters	a vector of strings, usually some letters such as "x" and "y", to denote the variables
collapse	a string to denote the symbol representing the multiplication, e.g. "*" or "."

**Details**

If the function returned by this function is applied to a vector of exponents whose length is higher than the length of the letters vector, then `showMonomialX1X2X3(x=letters[1])` is applied (see the last example).

**Value**

A function which takes as argument a sequence of exponents and which prints the corresponding monomial.

**Note**

The function returned by this function can be used as the option "showMonomial" in the `showQsprayOption<-` function.

**See Also**

[showQsprayXYZ](#), [showMonomialX1X2X3](#), [showQsprayOption<-](#).

**Examples**

```
showMonomialXYZ()(c(1, 0, 2))
showMonomialXYZ(collapse = "*")(c(1, 0, 2))
showMonomialXYZ()(NULL)
# what happens if there are more exponents than letters:
showMonomialXYZ(c("a", "b"), collapse = "*")(c(1, 2, 3))
# same as:
showMonomialX1X2X3("a", collapse = "*")(c(1, 2, 3))
# setting a show option:
set.seed(3141)
( qspray <- rQspray() )
showQsprayOption(qspray, "showMonomial") <- showMonomialXYZ(c("A", "B", "C"))
qspray
# this is equivalent to:
showQsprayOption(qspray, "showQspray") <- showQsprayXYZ(c("A", "B", "C"))
```

---

showQspray

*Print a 'qspray' object*

---

**Description**

Prints a qspray object given a function which prints the monomials.

**Usage**

```
showQspray(showMonomial, compact = FALSE, multiplication = "*")
```

**Arguments**

showMonomial	a function which takes as argument a sequence of exponents and which returns a string representing the corresponding monomial
compact	a Boolean value; if TRUE, then the + sign and the - sign will not be surrounded by spaces
multiplication	used to separate the coefficient and the monomial within a term

**Value**

A function which prints a qspray object.

**Note**

The function returned by this function can be used as the option "showQspray" in the [showQsprayOption<-](#) function. But one generally prefers to use [showQsprayX1X2X3](#) or [showQsprayXYZ](#) instead, which are both built with showQspray.

**See Also**

[showQsprayX1X2X3](#), [showQsprayXYZ](#), [showQsprayOption<-](#).

**Examples**

```
set.seed(3141)
( qspray <- rQspray() )
f <- showQspray(showMonomialX1X2X3("X"), compact = TRUE)
f(qspray)
# this is equivalent to:
f <- showQsprayX1X2X3("X", compact = TRUE)
f(qspray)
# if you want to adopt this way to show a qspray, use
# the setter function \link{showQsprayOption<-}:
showQsprayOption(qspray, "showQspray") <-
  showQsprayX1X2X3("X", compact = TRUE)
qspray
# then this show option will be preserved by some operations on the qspray:
qspray^2
```

---

`showQsprayOption<-`      *Set a show option to a 'qspray' object*

---

**Description**

Set a show option to a qspray object

**Usage**

```
showQsprayOption(x, which) <- value
```

**Arguments**

<code>x</code>	a qspray object
<code>which</code>	which option to set; this can be "x", "showMonomial", or "showQspray"
<code>value</code>	the value of the option

**Value**

This returns the updated qspray.

**Note**

The interest of setting some show options to a 'qspray' is that these options are preserved by some operations. See the examples and the README.

**Examples**

```

set.seed(3141)
( qspray <- rQspray() )
showQsprayOption(qspray, "x") <- "a"
qspray
# this is identical to:
showQsprayOption(qspray, "showMonomial") <- showMonomialX1X2X3("a")
# and also identical to:
showQsprayOption(qspray, "showQspray") <- showQsprayX1X2X3("a")
# old show method:
showQsprayOption(qspray, "showMonomial") <- showMonomialOld()
qspray
# show options are preserved by some operations:
qspray^2
3*qspray
derivQspray(qspray, 1)
swapVariables(qspray, 1, 2)
substituteQspray(qspray, c(NA, NA, "3/2"))
# for the binary arithmetic operations, the show options of the first
# operand are transferred to the result when possible:
( qspray2 <- rQspray() )
qspray + qspray2

```

---

<code>showQsprayX1X2X3</code>	<i>Print a 'qspray' object</i>
-------------------------------	--------------------------------

---

**Description**

Prints a qspray object given a string for the variable.

**Usage**

```
showQsprayX1X2X3(x = "x", collapse = ".", ...)
```

**Arguments**

<code>x, collapse</code>	see <a href="#">showMonomialX1X2X3</a>
<code>...</code>	arguments passed to <a href="#">showQspray</a> , such as <code>compact=TRUE</code>

**Value**

A function which prints a qspray object.

**Note**

The way qspray objects are displayed can be controlled with the help of the function [showQsprayOption<-](#), and `showQsprayX1X2X3()` is a possible option to pass in [showQsprayOption<-](#).

**See Also**

[showMonomialX1X2X3](#), [showQspray](#), [showQsprayOption<-](#).

**Examples**

```
set.seed(3141)
( qspray <- rQspray() )
showQsprayX1X2X3("X")(qspray)
# setting a show option:
showQsprayOption(qspray, "showQspray") <- showQsprayX1X2X3("A")
qspray
# this is equivalent to:
showQsprayOption(qspray, "showMonomial") <- showMonomialX1X2X3("A")
# and also equivalent to:
showQsprayOption(qspray, "x") <- "A"
```

---

showQsprayXYZ

*Print a polynomial*

---

**Description**

Prints a polynomial by printing monomials like "x^2.yz".

**Usage**

```
showQsprayXYZ(letters = c("x", "y", "z"), collapse = ".", ...)
```

**Arguments**

letters, collapse

see [showMonomialXYZ](#)

...

arguments passed to [showQspray](#), such as compact=TRUE

**Value**

A function which prints a qspray object. It is constructed with [showQspray](#) and [showMonomialXYZ](#).

**Note**

The function returned by this function can be used as the option "showQspray" in the [showQsprayOption<-](#) function.

**See Also**

[showMonomialXYZ](#), [showQspray](#), [showQsprayOption<-](#).

**Examples**

```

set.seed(3141)
( qspray <- rQspray() )
showQsprayXYZ(c("X", "Y", "Z"))(qspray)
showQsprayXYZ(c("X", "Y", "Z"))(qlone(1) + qlone(2) + qlone(3) + qlone(4))
# setting a show option:
showQsprayOption(qspray, "showQspray") <- showQsprayXYZ(c("A", "B", "C"))
qspray
# this is equivalent to:
showQsprayOption(qspray, "showMonomial") <- showMonomialXYZ(c("A", "B", "C"))

```

---

substituteQspray      *Substitutions in a 'qspray' polynomial*

---

**Description**

Substitute some variables in a qspray polynomial.

**Usage**

```
substituteQspray(qspray, values)
```

**Arguments**

qspray	a qspray object
values	the values to be substituted; this must be a vector whose length equals the number of variables of qspray, and whose each entry is either NA (for non-substitution) or a 'scalar' x such that as.character(x) is a quoted integer or a quoted fraction

**Value**

A qspray object.

**Examples**

```

library(qspray)
x <- qlone(1)
y <- qlone(2)
z <- qlone(3)
p <- x^2 + y^2 + x*y*z - 1
substituteQspray(p, c("2", NA, "3/2"))

```

---

swapVariables	<i>Swap variables</i>
---------------	-----------------------

---

**Description**

Swap two variables in a qspray polynomial.

**Usage**

```
## S4 method for signature 'qspray,numeric,numeric'  
swapVariables(x, i, j)
```

**Arguments**

x	a qspray object
i, j	indices of the variables to be swapped

**Value**

A qspray object.

**Examples**

```
library(qspray)  
f <- function(x, y, z) {  
  x^2 + 5*y + z - 1  
}  
x <- qlone(1)  
y <- qlone(2)  
z <- qlone(3)  
P <- f(x, y, z)  
Q <- swapVariables(P, 2, 3)  
Q == f(x, z, y) # should be TRUE
```

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