Package ‘lancor’

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Type Package

Title Statistical Inference via Lancaster Correlation

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Description
Implementation of the methods described in Holzmann, Klar (2024) <doi:10.1111/sjos.12733>.
Lancaster correlation is a correlation coefficient which equals the
absolute value of the Pearson correlation for the bivariate normal distribution,
and is equal to or slightly less than the maximum correlation coefficient
for a variety of bivariate distributions. Rank and moment-based estimators and
corresponding confidence intervals are implemented, as well as independence
tests based on these statistics.

Imports arrangements, boot, graphics, sn, stats

License GPL-2

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Suggests testthat (>= 3.0.0)

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NeedsCompilation no

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Contents

lcor *Lancaster correlation*

Description

Computes the Lancaster correlation coefficient.

Usage

```
lcor(x, y = NULL, type = c("rank", "linear"))
```

Arguments

x	a numeric vector, or a matrix or data frame with two columns.
y	NULL (default) or a vector with same length as x.
type	a character string indicating which lancaster correlation is to be computed. One of "rank" (default), or "linear": can be abbreviated.

Details

Let F_X and F_Y be the distribution functions of X and Y, and define

$$X^* = \Phi^{-1}(F_X(X)), \quad Y^* = \Phi^{-1}(F_Y(Y)),$$

where Φ^{-1} is the standard normal quantile function. Furthermore for X and Y with finite fourth moment, let

$$\tilde{X} = (X - \mathbb{E}(X))/\operatorname{sd}(X), \quad \tilde{Y} = (Y - \mathbb{E}(Y))/\operatorname{sd}(Y).$$

Then

$$\rho_L(X,Y) = \max\{|\operatorname{Cor}_{\text{Pearson}}(X^*,Y^*)|,\ |\operatorname{Cor}_{\text{Pearson}}((X^*)^2,(Y^*)^2)|\}$$

and

$$\rho_{L,1}(X,Y) = \max\{|\operatorname{Cor}_{\text{Pearson}}(X,Y)|,\ |\operatorname{Cor}_{\text{Pearson}}((\tilde{X})^2,(\tilde{Y})^2)|\}$$

are called the Lancaster correlation coefficient and the linear Lancaster correlation coefficient, respectively. Two estimation methods are supported:

- **Linear estimator for $\boldsymbol{\rho_{L,1}}$** (`type = "linear"`): Consider $\rho_{L1} = \operatorname{Cor}_{\text{Pearson}}(X,Y)$ and $\rho_{L2} = \operatorname{Cor}_{\text{Pearson}}((\tilde{X})^2,(\tilde{Y})^2)$. Let $\hat{\rho}_{L1}$ be the sample Pearson correlation and $\hat{\rho}_{L2}$ the empirical correlation of the squares of the empirically standardized observations, and set $\hat{\rho}_{L,1} = \max\{\,|\hat{\rho}_{L1}|,\ |\hat{\rho}_{L2}|\,\}$.
- **Rank-based estimator for $\boldsymbol{\rho_L}$** (`type = "rank"`): Consider $\rho_{R1} = \operatorname{Cor}_{\text{Pearson}}(X^*,Y^*)$ and $\rho_{R2} = \operatorname{Cor}_{\text{Pearson}}((X^*)^2,(Y^*)^2)$. Let Q_i and R_i be the ranks of X_i and Y_i, within $X_1,...,X_n$ or $Y_1,...,Y_n$ respectively. Define

$$\hat{\rho}_{R1} = \frac{1}{n\,s_a^2}\sum_{j=1}^{n} a(Q_j)\,a(R_j),$$

$$\hat{\rho}_{R2} = \frac{1}{n\, s_b^2} \sum_{j=1}^{n} \big(b(Q_j) - \bar{b}\big)\, \big(b(R_j) - \bar{b}\big),$$

where the scores are, for $j = 1, ..., n$,

$$a(j) = \Phi^{-1}\Big(\frac{j}{n+1}\Big), \quad b(j) = a(j)^2,$$

$$\bar{b} = \frac{1}{n}\sum_{j=1}^{n} b(j), \quad s_a^2 = \frac{1}{n}\sum_{j=1}^{n}\big(a(j) - \bar{a}\big)^2, \quad s_b^2 = \frac{1}{n}\sum_{j=1}^{n}\big(b(j) - \bar{b}\big)^2.$$

Finally, the rank-based Lancaster correlation is

$$\hat{\rho}_L = \max\big\{\, |\hat{\rho}_{R1}|, |\hat{\rho}_{R2}|\big\}.$$

Value

the sample Lancaster correlation.

Author(s)

Hajo Holzmann, Bernhard Klar

References

Holzmann, Klar (2024). "Lancester correlation - a new dependence measure linked to maximum correlation". doi:10.1111/sjos.12733

See Also

lcor.comp, lcor.ci, lcor.test

Examples

```
Sigma <- matrix(c(1,0.1,0.1,1), ncol=2)
R <- chol(Sigma)
n <- 1000
x <- matrix(rnorm(n*2), n)
lcor(x, type = "rank")
lcor(x, type = "linear")

x <- matrix(rnorm(n*2), n)
nu <- 2
y <- x / sqrt(rchisq(n, nu)/nu)
cor(y[,1], y[,2], method = "spearman")
lcor(y, type = "rank")
```

lcor.ci *Confidence intervals for the Lancaster correlation coefficient*

Description

Computes confidence intervals for the Lancaster correlation coefficient. Lancaster correlation is a bivariate measures of dependence.

Usage

```
lcor.ci(
  x,
  y = NULL,
  conf.level = 0.95,
  type = c("rank", "linear"),
  con = TRUE,
  R = 1000,
  method = c("plugin", "boot", "pretest")
)
```

Arguments

`x`	a numeric vector, or a matrix or data frame with two columns.
`y`	NULL (default) or a vector with same length as x.
`conf.level`	confidence level of the interval.
`type`	a character string indicating which lancaster correlation is to be computed. One of "rank" (default), or "linear": can be abbreviated.
`con`	logical; if TRUE (default), conservative asymptotic confidence intervals are computed.
`R`	number of bootstrap replications.
`method`	a character string indicating how the asymptotic covariance matrix is computed if type ="linear". One of "plugin" (default), "boot" or "symmetric": can be abbreviated.

Details

Computes asymptotic and bootstrap-based confidence intervals for the (linear) Lancaster correlation coefficient ρ_L ($\rho_{L,1}$). For more details see `lcor`.

Asymptotic confidence intervals are derived under two cases (analogously for ρ_L; see Holzmann and Klar (2024)):

Case 1: If $|\rho_{L1}| \neq |\rho_{L2}|$, the $1-\alpha$ asymptotic interval is

$$\left[\max\{\hat{\rho}_{L,1} - z_{1-\alpha/2}\, s/\sqrt{n}, 0\},\ \min\{\hat{\rho}_{L,1} + z_{1-\alpha/2}\, s/\sqrt{n}, 1\}\right],$$

where $z_{1-\alpha/2}$ is the standard normal quantile and s is an estimator of the corresponding standard deviation.

Case 2: If $|\rho_{L1}| = |\rho_{L2}| = a > 0$, let τ denote the correlation between the two components and let $q_{1-\alpha/2}$ be the $1-\alpha/2$ quantile of the asymptotic distribution of $\sqrt{n}(\hat{\rho}_{L,1} - a)$. A conservative asymptotic interval is

$$\left[\max\{\hat{\rho}_{L,1} - q_{1-\alpha/2}/\sqrt{n}, 0\},\ \min\{\hat{\rho}_{L,1} + z_{1-\alpha/2}\, s/\sqrt{n}, 1\}\right].$$

Additionally, bootstrap-based intervals can be obtained by resampling and estimating the covariance matrix of the rank or linear correlation components.

Value

a vector containing the lower and upper limits of the confidence interval.

Author(s)

Hajo Holzmann, Bernhard Klar

References

Holzmann, Klar (2024). "Lancester correlation - a new dependence measure linked to maximum correlation". doi:10.1111/sjos.12733

See Also

lcor, lcor.comp, lcor.test

Examples

```
n <- 1000
x <- matrix(rnorm(n*2), n)
nu <- 2
y <- x / sqrt(rchisq(n, nu)/nu) # multivariate t
lcor(y, type = "rank")
lcor.ci(y, type = "rank")
```

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lcor.comp *Lancaster correlation and its components*

Description

Computes the Lancaster correlation coefficient and its components.

Usage

```
lcor.comp(x, y = NULL, type = c("rank", "linear"), plot = FALSE)
```

Arguments

- `x` — a numeric vector, or a matrix or data frame with two columns.
- `y` — NULL (default) or a vector with same length as x.
- `type` — a character string indicating which lancaster correlation is to be computed. One of "rank" (default), or "linear": can be abbreviated.
- `plot` — logical; if TRUE, scatterplots of the transformed x and y values and of their squares are drawn.

Details

For more details see `lcor`.

Value

a vector containing the two components rho1 and rho2 and the sample Lancaster correlation.

Author(s)

Hajo Holzmann, Bernhard Klar

References

Holzmann, Klar (2024). "Lancester correlation - a new dependence measure linked to maximum correlation". doi:10.1111/sjos.12733

See Also

`lcor`, `lcor.comp`, `lcor.test`

Examples

```
Sigma <- matrix(c(1,0.1,0.1,1), ncol=2)
R <- chol(Sigma)
n <- 1000
x <- matrix(rnorm(n*2), n)
nu <- 8
y <- x / sqrt(rchisq(n, nu)/nu) #multivariate t
cor(y[,1], y[,2])
lcor.comp(y, type = "linear")

x <- matrix(rnorm(n*2), n)
nu <- 2
y <- x / sqrt(rchisq(n, nu)/nu) #multivariate t
cor(y[,1], y[,2], method = "spearman")
lcor.comp(y, type = "rank", plot = TRUE)
```

lcor.test *Lancaster correlation test*

Description

Lancaster correlation test of bivariate independence. Lancaster correlation is a bivariate measures of dependence.

Usage

```
lcor.test(
  x,
  y = NULL,
  type = c("rank", "linear"),
  nperm = 999,
  method = c("permutation", "asymptotic", "symmetric")
)
```

Arguments

`x`	a numeric vector, or a matrix or data frame with two columns.
`y`	NULL (default) or a vector with same length as x
`type`	a character string indicating which lancaster correlation is to be computed. One of "rank" (default), or "linear": can be abbreviated.
`nperm`	number of permutations.
`method`	a character string indicating how the p-value is computed if type ="linear". One of "permutation" (default), "asymptotic" or "symmetric": can be abbreviated.

Details

For more details on the testing procedure see $Remark$ 2 in Holzmann, Klar (2024).

Value

A list containing the following components:

`lcor`	the value of the test statistic
`pval`	the p-value of the test

Author(s)

Hajo Holzmann, Bernhard Klar

References

Holzmann, Klar (2024). "Lancester correlation - a new dependence measure linked to maximum correlation". doi:10.1111/sjos.12733

See Also

lcor, lcor.comp, lcor.ci and for for performing an ACE permutation test of independence see acepack (https://cran.r-project.org/package=acepack).

Examples

```
n <- 200
x <- matrix(rnorm(n*2), n)
nu <- 2
y <- x / sqrt(rchisq(n, nu)/nu)
cor.test(y[,1], y[,2], method = "spearman")
lcor.test(y, type = "rank")
```

Sigma.est — *Covariance matrix of components of Lancaster correlation coefficient*

Description

Estimate of covariance matrix of the two components of Lancaster correlation. Lancaster correlation is a bivariate measures of dependence.

Usage

```
Sigma.est(xx)
```

Arguments

xx	a matrix or data frame with two columns.

Details

For more details see the Appendix in Holzmann, Klar (2024).

Value

the estimated covariance matrix.

Author(s)

Hajo Holzmann, Bernhard Klar

References

Holzmann, Klar (2024). "Lancester correlation - a new dependence measure linked to maximum correlation". doi:10.1111/sjos.12733

See Also

lcor.ci

Examples

```
Sigma <- matrix(c(1,0.1,0.1,1), ncol=2)
R <- chol(Sigma)
n <- 1000
x <- matrix(rnorm(n*2), n)
nu <- 8
y <- x / sqrt(rchisq(n, nu)/nu) #multivariate t
Sigma.est(y)
```

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