Package 'ExtrPatt'

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Type Package

Title Spatial Dependencies and Indices for Extremes

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Description An implementation of
1) the tail pairwise dependence matrix (TPDM) as described in Jiang & Cooley (2020) <doi:10.1175/JCLI-D-19-0413.1>
2) the extremal pattern index (EPI) as described in Szemkus & Friederichs ('Spatial patterns and indices for heatwave and droughts over Europe using a decomposition of extremal dependency'; submitted to ASCMO 2023).

Depends R (>= 3.5.0)

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R topics documented:

`compute.EPI` *Estimation of EPI*

Description

Estimates the extremal pattern index (EPI) from either the 'm' principle components after a PCA or left- and right expansion coefficients after an SVD. In case of a SVD, the threshold-based EPI (TEPI) can optionally be calculated.

Usage

```
compute.EPI(coeff, m = 1:10, q = 0.98)
```

Arguments

`coeff`	A list, containing the t x n dimensional principle components/expansion coefficients of TPDM. Can also be output of function 'est.tpdm'.
`m`	numeric vector: Containing the Principle Components from which EPI shall be computed (e.g. with modes = c(1:10), the EPI is calculated on first ten principle components)
`q`	Optional: A threshold for computation of TEPI

Details

Given the first 'm' modes of principle components u and eigenvalues after a PCA, the EPI is given as:

$$EPI_t^u = \sqrt{\sum_{k=1}^{m}(u_{t,k}^2)/\sum_{j=1}^{m}e_j}.$$

Given the first 'm' modes of expansion coefficients u and v and singular values e after a SVD, the EPI and TEPI are given as:

$$EPI_t^{u,v} = \sqrt{\sum_{k=1}^{m}(u_{t,k}^2 + v_{t,k}^2)/\sum_{j=1}^{m}e_j}.$$

$$TEPI_t^{u,v} = \sqrt{(\sum_{k=1}^{m}(u_{t,k}^2 + v_{t,k}^2)/\sum_{j=1}^{m}e_j)|_{(|u_{t,k}|>q_u,|vt,k|>q_v)}}.$$

Value

An array of length t, containing EPI. TEPI is computed if if q > 0.

References

Szemkus & Friederichs (2023)

Examples

```
data    <- precipGER

data.alpha2  <- to.alpha.2(data$pr)
Sigma   <- est.tpdm(data.alpha2,anz_cores =1)
res.pca <- pca.tpdm(Sigma, data.alpha2)
EPI <- compute.EPI(res.pca, m = 1:10)

plot(data$date, EPI, type='l')
```

`decls` *Declustering*

Description

Declustering routine, which will can be applied on radial component r in estimation of the TPDM. Subroutine of est.tpdm.

Usage

```
decls(x, th, k)
```

Arguments

`x`	Real vector
`th`	Threshold
`k`	Cluster length

Value

numeric vector of declustered threshold exceedances

Author(s)

Yuing Jiang, Dan Cooley

References

Jiang & Cooley (2020) <doi:10.1175/JCLI-D-19-0413.1>

See Also

est.tpdm

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est.tpdm *Estimation of TPDM*

Description

Estimation of tail pairwise dependence matrix (TPDM)

Sub-Routine of est.row.tpdm. Calculates one element of the TPDM

Usage

```
est.tpdm(X, Y = NULL, anz_cores = 1, clust = NULL, q = 0.98)

est.row.tpdm(x, Y, clust = NULL, q = 0.98)

est.element.tpdm(x, y, clust = NULL, q = 0.98)
```

Arguments

X	A t x n dimensional, numeric data-matrix with t: Number of time steps and n: Number of grid points/stations
Y	A t x n dimensional, numeric Data-matrix with t: Number of time steps and n: Number of grid points/stations
anz_cores	Number of cores for parallel computing (default:1); Be careful not to overload your computer!
clust	Optional: If clust = NULL, no declustering is performed. Else, declustering according to cluster-length 'clust'.
q	Threshold for computation of TPDM. Only data above the 'q'-quantile will be used for estimation. Choose such that 0<q<1.
x	Array of length t, where t is the number of time steps
y	Same as x

Details

Given a random vector X with components $x_{t,i}, x_{t,j}$ with $i, j = 1, \ldots, n$ and it's radial component $r_{t,ij} = \sqrt{x_{t,i}^2 + x_{t,j}^2}$ and angular components $w_{t,i} = x_{t,i}/r_{t,ij}$ and $w_{t,j} = x_{t,j}/r_{t,ij}$, the i'th,j'th element of the TPDM is estimated as:

$$\hat{\sigma}_{ij} = 2n_{ij,exc}^{-1} \sum_{t=1}^{n} w_{t,i} w_{t,j}|_{(r_{t,ij} > r_{0,ij})}$$

. Given two random vectors X and Y with components $x_{t,i}, y_{t,j}$ with $i, j = 1, \ldots, n$, and it's radial component $r_{t,ij} = \sqrt{x_{t,i}^2 + y_{t,j}^2}$ and angular components $w_{t,i}^x = \frac{x_{t,i}}{r_{t,ij}}; w_{t,j}^y = \frac{y_{t,j}}{r_{t,ij}}$, the i'th,j'th element of the cross-TPDM is estimated as:

$$\hat{\sigma}_{ij} = 2n_{exc}^{-1} \sum_{t=1}^{n} w_{t,i}^x w_{t,j}^y |_{(r_{t,ij} > r_{0,ij})}$$

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Value

An n x n matrix, containing the estimate of the TPDM

Array containing the estimate of one row of the TPDM.

Value containing the estimate of one element of the TPDM.

References

Jiang & Cooley (2020) <doi:10.1175/JCLI-D-19-0413.1>; Szemkus & Friederichs (2023)

Examples

```
data    <- precipGER

data.alpha2       <- to.alpha.2(data$pr)
Sigma   <- est.tpdm(data.alpha2,anz_cores =1)
```

`invTrans` *Transformation function*

Description

Applies the inverse transformation $t^{-1}(v) = \log(\exp{(v)} - 1)$

Usage

```
invTrans(v)
```

Arguments

`v` Real, positive vector

Details

Transformation from real, positive vector in real vector under preservation of frechet-distribution.

Value

Real vector, containing the result of inverse transformation function.

Author(s)

Yuing Jiang, Dan Cooley

References

Cooley & Thibaud (2019) <doi:10.1093/biomet/asz028>

See Also

svd.tpdm, pca.tpdm

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pca.tpdm *Principal Component Analysis for TPDM*

Description

Calculates principal component analysis (PCA) of given TPDM

Usage

```
pca.tpdm(Sigma, data)
```

Arguments

`Sigma`	A n x n data array, containing the TPDM, can be output of est.tpdm.
`data`	A t x n dimensional, numeric Data-matrix with t: Number of time steps and n: Number of grid points/stations.

Value

list containing

- pc: The Principal Components of TPDM
- basis: The Eigenvectors of TPDM
- extremal.basis: The Eigenvectors of TPDM but transformed in positive reals with trans

Author(s)

Yuing Jiang, Dan Cooley

References

Jiang & Cooley (2020) <doi:10.1175/JCLI-D-19-0413.1>

`precipGER` *daily Precipitation over Southern Germany*

Description

Daily Precipitation at several stations in Germany

Usage

```
data(precipGER)
```

Format

A list containing containing

- pr: data-array
- date: time-information
- lon,lat: longitude & latitude information

Details

Daily Precipitation Data

Daily precipitation data from several wather station in southern Germany (longitude <50) over the years 2000-2019. The data has been downloaded from opendata server of german weather service (https://opendata.dwd.de/climate_environment/CDC/observations_germany/climate/daily/kl/historical/).

Source

Quelle: Deutscher Wetterdienst

`svd.tpdm` *Singular Value decomposition for cross-TPDM*

Description

Calculates singular value decomposition (SVD) of given cross-TPDM

Usage

```
svd.tpdm(Sigma, X, Y)
```

Arguments

`Sigma`	A n x n data array, containing the cross-TPDM, can be output of est.tpdm.
`X`	A t x n dimensional, numeric Data-matrix with t: Number of time steps and n: Number of grid points/stations.
`Y`	Same as X but for second variable.

Value

List containing

- pcU, pcV: The left- and right expansion coefficients of cross-TPDM
- U, V: The left- and right singular Vectors of cross-TPDM
- extr.U, extr.V: The left- and right singular vectors of cross-TPDM, but transformed in positive reals with trans

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`to.alpha.2` *Probability integral transformation*

Description

Performs transformation to make all of the margins follow a Frechet distribution with tail-index alpha = 2.

Usage

```
to.alpha.2(data, orig = NULL)
```

Arguments

`data`	A t x n dimensional, numeric Data-matrix with t: Number of time steps and n: Number of grid points/stations
`orig`	If known: original distribution of data (currently implemented: 'normal' or 'gamma'), else: NULL

Value

Data-matrix of same dimension as 'data', but in Frechet-margins with tail-index 2

`trans` *transformation function*

Description

Applies the transformation $t(x) = \log(1 + \exp{(x)})$

Usage

```
trans(x)
```

Arguments

`x`	Real vector

Details

Transformation from real vector in real, positive vector under preservation of Frechet-distribution.

Value

Real, positive vector, containing the result of transformation function.

Author(s)

Yuing Jiang, Dan Cooley

References

Cooley & Thibaud (2019) <doi:10.1093/biomet/asz028>

See Also

svd.tpdm, pca.tpdm

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wrapper.EPI *Wrapper function*

Description

Handles all steps for estimation of EPI from raw-data: 1) Preprocessing into Frechet-Margins 2) Estimation of TPDM 3) Calculation of Principal Components 4) Estimation of EPI

Usage

```
wrapper.EPI(
  X,
  Y = NULL,
  q = 0.98,
  anz_cores = 1,
  clust = NULL,
  m = 1:10,
  thr_EPI = NULL
)
```

Arguments

X	A t x n dimensional Data-matrix with t: Number of time steps and n: Number of grid points/stations
Y	Optional: Sames as X but for second variable: If Y!=NULL, cross-TPDM instead of TPDM and SVD instead of PCA is computed
q	Threshold for computation of TPDM. Only data above the 'q'-quantile will be used for estimation. Choose such that $0 < q < 1$.

`anz_cores`	Number of cores for parallel computing (default: 5)
`clust`	Optional_ Uf clust = NULL, no declustering is performed. Else, declustering according to cluster-length 'clust'
`m`	Numeric vector: Containing the principal components/expansion coefficients (in case of Y!=NULL) from which the EPI shall be computed (default: modes = c(1:10), calculates the EPI on first ten principle Components)
`thr_EPI`	Only if Y!=NULL: Threshold for computation of TEPI. Expansion-coefficients that exceed the 'q'-quantile will be used for estimation. Choose such that 0 < q < 1.

Value

In case of Y =NULL: A list containing:

- basis: The Eigenvectors of TPDM
- pc: The principal components of TPDM
- extremal.basis: The Eigenvectors of TPDM but transformed in positive reals with trans
- EPI: Extremal pattern index

In case of Y !=NULL: A list containing:

- U, V: The left- and right singular Vectors of cross-TPDM
- extr.U, extr.V: The left- and right singular vectors of cross-TPDM, but transformed in positive reals with trans
- pcU, pcV: The left- and right expansion coefficients of cross-TPDM
- EPI: Extremal pattern index
- TEPI: Threshold-based extremal pattern index

References

Szemkus & Friederichs 2023

Examples

```
data    <- precipGER

result  <- wrapper.EPI(data$pr, m = 1:50)

rbPal <- colorRampPalette(c('blue', 'white','red'))
Col <- rbPal(10)[as.numeric(cut(result$basis[,2],breaks = 10))]
plot(data$lat, data$lon,col=Col)
plot(data$date, result$EPI, type='l')
```

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